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A NOVEL NUMERICAL TOPOLOGY OPTIMIZATION METHOD

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ABSTRACT

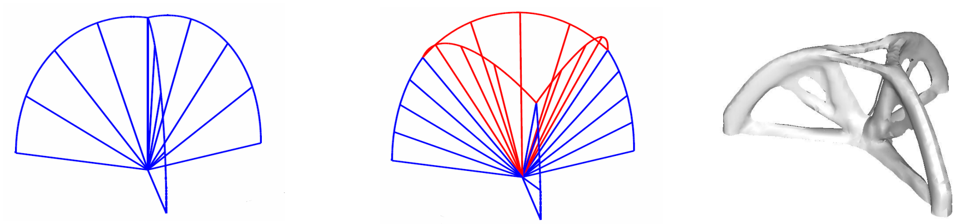
In recent years, finite element based topology optimization techniques have been developed and applied to the design of minimum weight structures. Examples of well known schemes include homogenization [1], Evolutionary Structural Optimization [2], and biologically inspired growth methods [3]. While the effectiveness of these methods has been well documented, difficulties associated with mathematical complexity, numerical inefficiencies and numerical instabilities have also been noted. A recent investigation at the University of Rhode Island has led to the development of a novel finite element based optimization scheme implemented through a series of user subroutines in a commercial finite element package. This implementation provides a robust design tool that has been shown to successfully identify minimum weight topologies for a variety of two and three-dimensional problems.

The novel implementation is based on material redistribution in which a variable density field is imposed at each finite element iteration. While maintaining a constant total mass of the structure, material is redistributed by increasing the density in regions of high strain energy and decreasing the density in regions of low strain energy. The density and associated material stiffness distributions within each element are imposed through interpolation of nodal values to the element Gauss points. Careful selection of an appropriate family of cumulative probability distribution functions to determine nodal densities at each iteration provides a gradual transition from a unimodal initial density distribution to a final bimodal distribution. The final density distribution provides the optimal topology in which fully dense and nearly void regions are clearly defined.

Initial trials have indicated that optimal topologies for several two and three-dimensional problems are accurately identified in relatively few finite element iterations. Furthermore, due to the imposition of continuous density variation within each element, numerical instabilities are not observed. Examples of several demonstration cases will be included in the conference presentation.

As one demonstration of the method, a family of 3-dimensional structures is analyzed based on the Michell semi-circular arch beam. These simple truly 3-dimensional structures comprise support points around a pitch circle with a vertical load applied at the center. For simplicity of the discussion, three equispaced supports will be used. If

the allowable structure height can be equal to or greater than the pitch circle radius of the supports, then a structure comprising three 90 degree centered fans is a globally optimal analog of the Michell 2-D arch. This structure is illustrated in Figure 1(a).



(a) (b) (c)

Figure 1 Tripod Michell Arch Structures

A simple height reduction modification does exist for the tripod arch structure. At any latitude, as the structure builds upwards from the support and load points, it is permissible to join the growing circular arches with three additional great circle arches to form a spherical triangle. The added circular arches, which form a crown to the structure, must then be supported by additional radial members reaching down to the loading point. Note that, in this case, the rotation of the horizontal radial members will now be greater than for the plane Michell arch.

One example of this family of structures is illustrated in Fig. 1(b). The two sets of arches are shown in different color to indicate that they are separate analytic regions, which together define a kinematically permissible displacement field. This makes the solutions only locally optimal with respect to a domain extending upwards to the level of the spherical triangle.

It can be shown that the volume of this Michell tripod structure is given by

$$V_{tripod} = 2Fa(\theta + \beta / \cos(\alpha / 2)) / \sigma$$

where F = supported center load, a = circle radius, θ = angle subtended by side arches, α = angle of equilateral upper spherical triangle, β = angle subtended by upper arches, and σ = material strength. It is evident that this volume converges rapidly to the global optimal volume as θ tends to $\pi/2$.

This tripod structure presents a challenge for structural topological optimization methods. A close approximation, using the three lower arches, but terminating at a defined angle θ with a plane triangular truss is an attractive alternative target, e.g. Fig. 1(c). It can be shown however that this always has a larger volume to support the same load.

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