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A MESO-SCALE APPROACH FOR THE NONLINEAR ANALYSIS OF ANGLE-PLY LAMINATES WITH DAMAGE

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ABSTRACT

A meso-scale approach for analysis of angle-ply fibrous composite laminates is presented. Three-dimensional two-cell representations of laminates with periodic fibre arrays are established and the corresponding periodic boundary conditions are derived. A nonlinear viscoelastic model developed by the authors is used to describe the constitutive behaviour of the matrix, and a smeared crack method is adopted to model the initiation and evolution of matrix damage. As an application of this unified approach, numerical predictions of the response of glass fibre/epoxy angle-ply laminates are presented. Using the material constants of the constituents, the global stress-strain response and the evolution of matrix damage of the angle-ply laminates are predicted.

Figure 1a shows a meshed three dimensional meso-scale repeated-cells (RC) representation of a $\pm\theta$ angle-ply laminate [1]. It consists of two rhombus prism units; each unit containing a single fibre represented by the circular cylindrical part in Fig.1a.

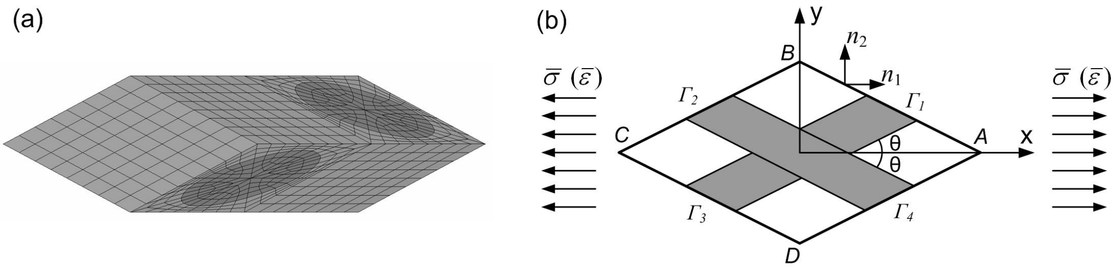

Fig.1 Meso-scale representation of an angle-ply laminate and the loading scenario

To apply the global stress or strain to the RC as shown schematically in Fig.1b, a new scheme has been proposed for specifying periodic boundary conditions (PBCs) of the RC model in finite element analysis. Expressed by the displacement components of master nodes, PBCs can be reduced to nodal displacement constraint equations on opposite boundary surfaces. Furthermore, the global loading conditions, either in the form of global stress or global strain or combinations thereof, can be easily applied by specifying the displacements or forces of the corresponding master nodes.

Maximum principal strain is used as the crack initiation criterion. At each matrix integration point, a crack coordinate system (1-2-3) is established in which the three axes are aligned with the three principal strains, $\varepsilon_1 \geq \varepsilon_2 \geq \varepsilon_3$ (see Fig. 2a). When ε_1^{max}

is greater than a critical value, a crack is assumed to form in the 2-3 plane. It is further assumed that a crack cannot transfer normal and shear stresses across the crack plane, i.e., σ_{11}, σ_{12} and $\sigma_{13} \rightarrow 0$. However, the ability to transfer the other stress components is not affected [2]. The evolution of the crack is described by a post-damage softening stress-strain relationship as in the following matrix form:

$$\{\Delta\sigma\}^{cr} = E_t[D]\{\Delta\varepsilon\}^{cr} - \chi[B]\{\sigma\}^{cr} \tag{1}$$

In Eqn.(1) E_t is the modulus of the epoxy under uniaxial tensile loading at the instant of damage initiation. [D] is a matrix determined by the Poisson ratio and a small number β which represents the loss of the stiffness in the three stress directions. The choice of a constant matrix [B] and a scalar constant χ allows the three stress components to decay to near-zero values in a sufficiently short duration [2].

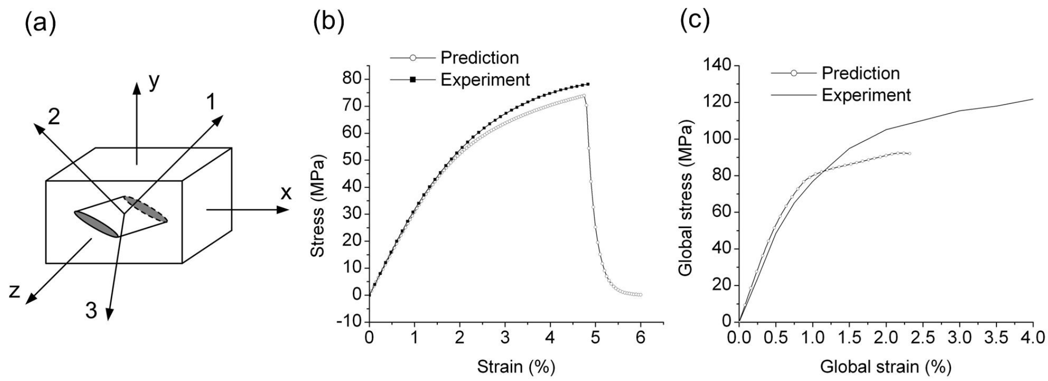

Fig. 2 (a) Crack coordinate system; (b) response of a single matrix element under uniaxial loading; (c) response of a $\pm 45°$ angle-ply laminates under uniaxial loading

Using a viscoelastic material model, post-damage model and the constants given in [2], Fig. 2b shows the response of a matrix material element before and after damage formation. Before damage initiation, the prediction is in good agreement with the test results which shows significant nonlinear response. After damage initiation, stress drops rapidly to zero, thus simulating the damage process.

As an application example, the uniaxial loadings of $\pm\theta$ angle-ply laminates with different angle values are analyzed using the finite element method. The global strain rate applied to the laminates is 10^{-4}/s. The composite system studied is E-glass fibre reinforced epoxy with a fibre volume fraction of 52.5%. Fig. 2c shows the predicted stress-strain curve of the $\pm 45°$ laminates along with the test data[2]. Examination of the predictions of macro and meso scale responses indicates that the apparent ‘yielding’ in the stress-strain curve, Fig 2c, is related to the initiation and propagation of matrix-cracking within the composite laminates.

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