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A multi-level fast multipole multi-region method for 3D seismic response of alluvial basins

*** S. Chaillat[1,2], M. Bonnet[1] and J.F. Semblat[2]**

[1] Solid Mechanics Laboratory
École Polytechnique
91128 Palaiseau Cedex, France
{bonnet,chaillat}@lms.polytechnique.fr

[2] Université Paris-Est, LCPC
58 bd Lefebvre
75015 Paris, France
semblat@lcpc.fr

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ABSTRACT

To compute seismic wave propagation in alluvial basins, various numerical methods have been proposed: the finite element method, the spectral element method, the finite difference schemes, the finite volume approaches and the boundary element method (BEM: [2]). The latter is well suited to the computation of seismic wave propagation in that only the domain boundaries (and possibly interfaces) are discretized, leading to a reduced number of degrees of freedom (DOFs). In traditional BE implementation the dimensional advantage with respect to domain discretization methods is offset by the fully-populated nature of the BEM coefficient matrix, with set-up and solution times rapidly increasing with the problem size N. The solution step may be performed using either direct solvers (typically based on a LU factorization) or iterative solvers (typically GMRES). The solution time is $O(N^3)$ for the former, and $O(N^2)$ per iteration for the latter. Since the overall number of iterations is usually much smaller than N, iterative solvers are preferable for large BEM models. Because of the large memory costs, large N ($N > 10^4$ on a single proc. PC) prohibit the use of traditional BEM implementation. As a result, the resolution of realistic seismological problems (geometry complexity, heterogeneity) is limited by the number of DOFs that can be solved on a given computer. Moreover, because the problem is solved in the frequency domain, the mesh size is linked to the problem characteristic wavelength. Consequently, the frequency range is also limited by the use of the traditional BEM.

Methodology. Each GMRES iteration requires the computation of one matrix-vector product, hence the $O(N^2)$ computing time since the matrix is full. In other areas where the BEM is used (electromagnetism, acoustics,...), considerable speedup of solution time and decrease of memory requirements have been achieved through the development, over the last decade, of the Fast Multipole Method (FMM: [3], [5]). The goal of the FMM is to speed up the matrix-vector product computation. This is achieved by (i) using a multipole expansion of the relevant Green's tensor, which (unlike in the standard BEM) allows to re-use element integrals for all collocation points, and (ii) defining a (recursive, multi-level) partition of the region of space enclosing the domain boundary of interest into cubic cells, allowing to optimally cluster influence computations according to the ratio between cluster size and distances between two such clusters. Moreover, the governing matrix is never explicitly formed, which leads to

a storage requirement well below the $O(N^2)$ memory necessary for holding the complete matrix. The FMM-accelerated BEM therefore achieves substantial savings in both CPU time and memory.

Outline and results. In this work, the FMM is extended to the 3-D frequency-domain elastodynamics and applied to the computation of seismic wave propagation in 3-D [1], a field of application addressed in only a few other references [4]. This communication is organized as follows. First, the main features of the elastodynamic FMM-BEM formulation are concisely presented. Then, numerical efficiency and accuracy are assessed on the basis of numerical results obtained for problems having known solutions (performed on a single PC computer for problem sizes of up to $N = O(10^6)$). In particular, numerical results are in agreement with the expected theoretical complexity of the FMM-accelerated elastodynamic BEM. The BEM formulation presented here uses the fundamental solutions of the full space. As a result, to study the propagation of seismic waves in alluvial basins, it is necessary to use a BE-BE coupling. The strategy used and results obtained for multi-material (piecewise homogeneous) media is presented. Finally, the present FMM-BEM is demonstrated on seismology-oriented examples, namely the study of the diffraction of a plane wave by a canyon or an alluvial basin. The influence of the size of the meshed part of the free surface is studied, and computations are performed for nondimensional frequencies higher than those considered in other studies, with which comparisons are made whenever possible. Ongoing research includes the formulation of a multipole expansion for the half-space elastodynamic fundamental solution.

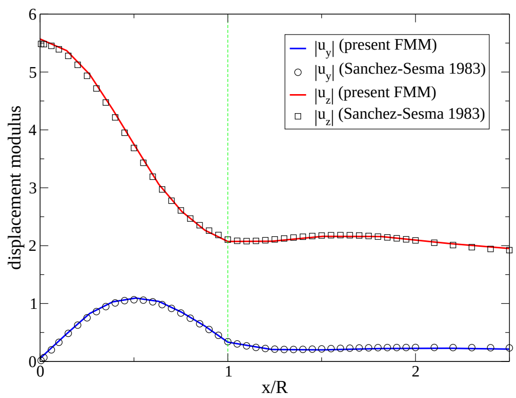

Figure 1: Diffraction of an incident P plane wave by a semi-spherical alluvial basin of radius R: horizontal and vertical computed displacement on the free surface plotted against normalized abscissa y/R (normalized frequency $\eta_P = k_P/R = 0.5$). Comparison of present FMM solution to results from Sánchez-Sesma [6].

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