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On Topology Optimization With Vanishing Constraints

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ABSTRACT

We consider optimization problems with constraints which must be ignored at certain points of the feasible domain. Such problems we call Mathematical Programs with Vanishing Constraints (MPVC). Our main application field of MPVCs is topology optimization of structures subject to stress constraints, but also buckling constraints etc. are covered by the framework of MPVCs. Typically, after finite element discretization (respectively, for discrete structures), these problems can be stated as an optimization problem in the following way:

$$\begin{array}{lrcll} & \min\limits_{\mathbf{x},\mathbf{u}} f(\mathbf{x},\mathbf{u}) & & & \\ \text{s.t.} & \mathbf{K}(\mathbf{x})\mathbf{u}_\ell & = & \mathbf{p}_\ell & \text{for } \ell = 1,\dots,L \\ & h_i(\mathbf{x},\mathbf{u}) & = & 0 & \text{for } i = 1,\dots,m_1 \\ & g_i(\mathbf{x},\mathbf{u}) & \le & 0 & \text{for } i = 1,\dots,m_2 \\ & x_i\, G_i(\mathbf{x},\mathbf{u}) & \le & 0 & \text{for } i = 1,\dots,n \\ & x_i & \ge & 0 & \text{for } i = 1,\dots,n \end{array} \tag{P}$$

Here the design variable is denoted by $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u}_\ell$ is the vector of displacements for load case ℓ, and $K(\mathbf{x})$ denotes the stiffness matrix, as usual. Notice that $\mathbf{K}(\mathbf{x})$ may well be singular for designs $\mathbf{x}$ with many zero components (topology problem!). The objective function f may denote weight, compliance etc. The functions h_i and g_i model ordinary side-constraints such as compliance bounds, a volume restriction, or others. The main issue of the problem lies in the constraints of the type "$x_i\, G_i(\mathbf{x},\mathbf{u}) \le 0$" which we call "vanishing constraints". Notice that each of these constraints is equivalent to the expression "$G_i(\mathbf{x},\mathbf{u}) \le 0$ if $x_i > 0$". Hence, the inequality constraint "$G_i(\mathbf{x},\mathbf{u}) \le 0$" *vanishes from the problem* if $x_i = 0$.

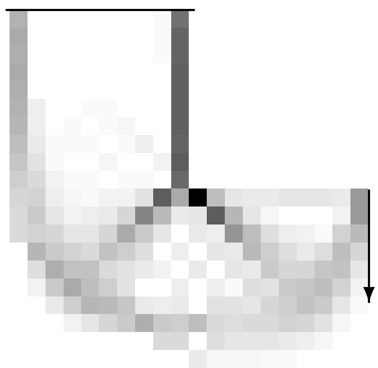
Vanishing constraints are required, e.g., for the exact modelling of stress constraints in topology optimization. It is essential to consider stress constraints just for those elements in the structure containing material. Similar comments can be made for the modelling of other local constraints relying on a "mechanical response", i.e., on $x_i > 0$.

Problem (P) is a problem in standard form of Nonlinear Programming. It turns out, however, that standard algorithms (generally) do not work very well and often fail or, even worse, produce erroneous results. The reasons for this bad behaviour can be explained by theory. It turns out that MPVCs possess bad properties concerning optimality conditions, namely, standard constraint qualifications are unlikely to hold at local minimizers. As a consequence, it may happen that at optimal points the standard first order optimality conditions do not hold. This pitfall similarly occurs for the well known problem class of Mathematical Programs with Complementarity Constraints (MPCC).

There are several options to attack this difficulty. **1.** Standard optimality conditions can be applied in a small neighbourhood of a local minimizer (see [1]). **2.** Similarly to MPCCs, adapted optimality conditions and corresponding constraint qualifications can be developed for the class of MPVCs (see [2,3,5]). **3.** By the introduction of auxiliary variables, each MPVC can be formulated as an MPCC (see [2,3,4,5]). It is then possible to treat MPVCs as MPCCs (see [4,5]). **4.** A perturbed problem is treated. It is then necessary to investigate the behaviour of solutions in relation to solutions of the unperturbed problem.

The talk discusses some new theoretical results on MPVCs and presents the outcome of some numerical experiments. As test examples we consider topology optimization problems with stress constraints.

Consider the famous example with an L-shaped domain and a single vertical point load applied at its upper right hand corner. The domain is discretized by 300 square elements. We treat a minimum weight problem subject to constraints on von-Mises stress formulated by vanishing constraints. For the calculated structure shown to the right 47 out of 300 stress constraints are "properly vanishing". This means, at the calculated point $(\mathbf{x}^*, \mathbf{u}^*)$ in (P) we have $G_i(\mathbf{x}^*, \mathbf{u}^*) > 0$ and $x_i^* = 0$ for 47 indices (= elements) i. Notice that $(\mathbf{x}^*, \mathbf{u}^*)$ is infeasible if the stress constraints are not modelled as vanishing constraints.



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