8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Finite Element Method in Solution of Elastic Waves Problems on Domains with Multi-Scale Feature

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KEY WORDS: *Unbounded domain, Multi-scale, Discrete Green's functions, Periodic material.*

ABSTRACT

Multi-scale modeling of problems with hierarchical material properties has recently been the focus of many scientists. The idea stems from the growing need for determining the material behaviour, at structural scale, by taking into account the properties of the finer scales of its structure. The key point of the modellong is the strategy used for bridging between the scales of the multi-scale system.
In this research, we shall address a model for bridging between the scales of two-scale materials, e.g. materials with macro and meso scales. In our study we shall consider a periodic structure for the finer scale of the material. Such a periodicity is extensively used in homogenization approaches. The so constructed systems, at the fine scale, may be first modelled as an unbounded domain and then the effects of the boundaries may be taken into account by a consistent boundary integral approach. Therefore the problem of concern in this work is reduced to behaviour study of an unbounded domain with periodic material properties. Here we shall focus on propagation of harmonic waves in such domains. The study may be performed by finding Green's functions on unbounded domains. However, such Green's functions are not generally available for domains with periodic material. Therefore numerically evaluated Green's functions, or discrete ones, seem to be well suited for our purpose. For the evaluation of the discrete Green's functions, a singular source is applied at a generic point of the unbounded domain. The solution is to be found by applying the radiation conditions for the wave problem.

Dealing with so defined problem needs behaviour prediction of FEM when the entire unbounded domain is assumingly discretized with elements. A new approach has recently been introduced by the authors in [1,2] assuming that the entire unbounded domain is discretized into similar patterns of elements. The model is capable of solving problems with periodic material properties.

In the presentation, we shall show how one can extend the application of the method proposed in [2] to problems with periodic material properties. The extension of the theory needs some additional assumptions stemming from Bloch-Floquet theory for differential equations with periodic coefficients [3]. With the principles of such a theory the radiation condition is satisfied through the proposed method. Similar to the work in [1], we shall use a discrete transformation technique introduced by the authors [1,2] in order to satisfy the boundary conditions defined on

a quadrant of the unbounded domain.
The results have been compared with those obtained from the exact solution of an equivalent problem with homogenized material. Excellent agreement between the amplitudes and the wavelengths of the solutions shows the validity of the proposed method.

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