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An unsteady adaptive stochastic finite elements uncertainty quantification method for fluid-structure interaction problems

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ABSTRACT

An Adaptive Stochastic Finite Elements approach for unsteady problems is developed. Unsteady solutions of dynamical systems are known to be sensitive to small input variations. Stochastic Finite Elements methods [1] usually require a fast increasing number of elements with time to capture the effect of random input parameters in these time-dependent problems. The resulting large number of samples required for resolving the long-term asymptotic stochastic behavior, results for computationally intensive fluid-structure interaction simulations in impractically high computational costs. The Unsteady Adaptive Stochastic Finite Elements (UASFE) formulation proposed in this paper maintains a constant interpolation accuracy in time with a constant number of samples. The approach is based on a time-independent parametrization of the sampled time series in terms of frequency, phase, amplitude, reference value, damping, and higher-period shape function [2]. This parametrization is interpolated using a robust Adaptive Stochastic Finite Elements method based on Newton-Cotes quadrature in simplex elements [3]. This approach requires a relatively low number of deterministic solves and preserves monotonicity and optima of the samples. In order to ensure the robustness of the method, (1) the elements are refined adaptively until convergence is reached in the L_∞-norm, and (2) the parametrization error is computed to determine the time interval in which the UASFE approximation is valid.

Results for a mass-spring-damper system and the Duffing equation with multiple random input parameters are presented. For the mass-spring-damper system the effect of positive and negative damping on the stochastic results is studied, see Figure 1. Input randomness assumed in a combination of the spring stiffness parameter and the damping parameter shows that a non-zero probability of negative damping results asymptotically in a diverging output standard deviation. In case of these two random parameters, the required number of samples for a converged UASFE approximation is a factor $2.6 \cdot 10^3$ smaller than for Monte Carlo simulation. The study of two random initial conditions for the Duffing equation illustrates that nonlinear dynamical systems with discontinuous solutions can be extremely sensitive to random initial conditions, see Figure 2. An amplification factor of 52 has been observed for the standard deviation. In the paper, results for the stochastic bifurcation behavior of a two-degree-of-freedom elastically mounted airfoil will also be presented.

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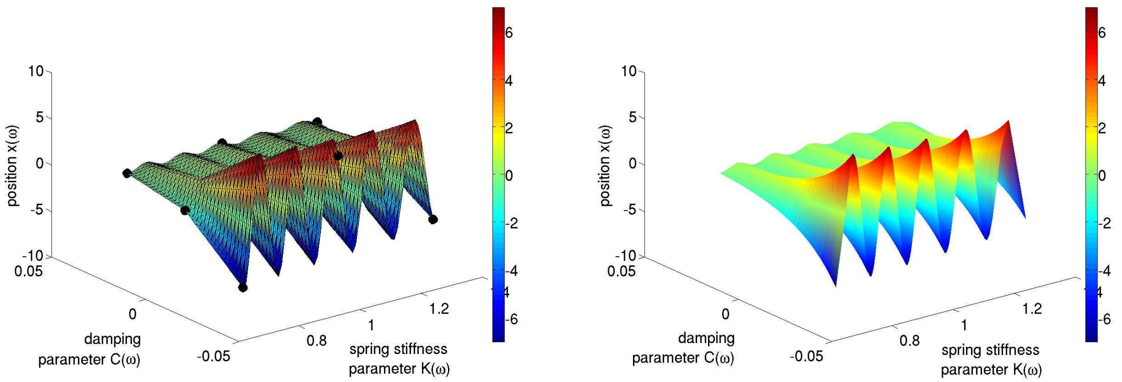
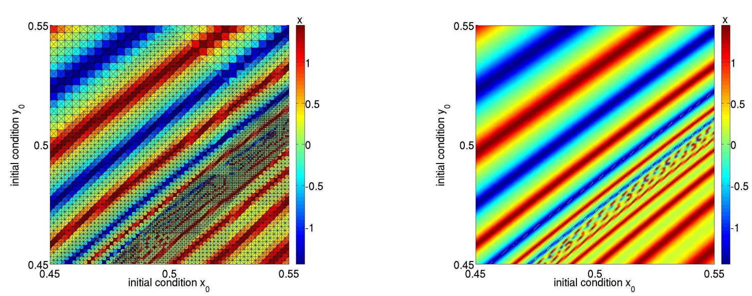


(a) UASFE, $N_{\mathrm{s}} = 9$ samples

(b) Monte Carlo, $N_{\mathrm{s}} = 1.2 \cdot 10^5$ samples

Figure 1: Two-dimensional response surface of $x(t,\omega)$ at $t_{\mathrm{stop}} = 100$ as function of the random stiffness $K(\omega)$ and damping $C(\omega)$ by two-dimensional Unsteady Adaptive Stochastic Finite Elements (UASFE) with $N_{\mathrm{e}} = 2$ ($N_{\mathrm{e_{sub}}} = 4^6$, $N_{\mathrm{s}} = 9$) and Monte Carlo (MC) simulation with $N_{\mathrm{s}} = 1.2 \cdot 10^5$ for the mass-spring-damper system.



(a) UASFE $x(t,\omega)$

(b) MC $x(t,\omega)$

Figure 2: Two-dimensional response surface approximations for $x(t_{\mathrm{stop}},\omega)$ as function of random initial conditions $x_0(\omega)$ and $y_0(\omega)$ by Unsteady Adaptive Stochastic Finite Elements (UASFE) with $N_{\mathrm{e}} = 64$ ($N_{\mathrm{e_{sub}}} = 4^4$, $N_{\mathrm{s}} = 151$) and Monte Carlo (MC) with $N_{\mathrm{s}} = 10^4$ for the Duffing equation.