**8th. World Congress on Computational Mechanics (WCCM8)**
**5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)**
**June 30 –July 5, 2008**
**Venice, Italy**

# Implicit A Posteriori Error Estimators for the Maxwell Equations

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**Key Words:** *Maxwell equations, finite element methods, a posteriori error estimation.*

## ABSTRACT

The numerical solution of the Maxwell equations, which describe the behavior electromagnetic waves, is complicated due to the limited regularity of the solution, such as near sharp and non-convex corners and at material interfaces. Resolving these local structures requires solution adaptive finite element techniques, which either locally refine the computational mesh or adjust the polynomial order. For the control of the adaptation process detailed knowledge about the error distribution is necessary, which can be provided by a posteriori error analysis. Compared to elliptic partial differential equations, the a posteriori error analysis of the Maxwell equations is considerably more complicated due to lack of regularity and the fact that the bilinear form is in general not coercive.

A posteriori error estimation techniques can be subdivided into explicit and implicit methods. For a general overview, see [1]. Explicit error estimators provide theoretical upper and lower bounds for the local error based on the numerical solution, but these techniques generally contain unknown coefficients and are frequently not sharp, see e.g. [2, 6] for an application to the Maxwell equations. In implicit error estimators an estimate for the numerical error is obtained by formulating local error equations, together with properly chosen boundary conditions, and solving these equations using finite element techniques. The main benefit of this approach is that no arbitrary coefficients occur in the error estimates, but this comes at the price of additional computational cost.

In this presentation we will give an overview of the implicit a posteriori error estimation techniques which we recently developed for the Maxwell equations [3, 4]. In this method higher order bubble functions and modified Nédélec basis functions are used to solve the local error equations on hexahedral and tetrahedral elements, respectively. The discrete weak formulation satisfies an inf-sup conditions which ensures the well posedness of the error equations. The estimated error distribution correlates well with the actual error distribution and is used to adapt the computational mesh.

An important issue in a posteriori error estimation is the reliability and efficiency of the error indicator. Recently, the a posteriori error estimators for the Maxwell equations developed in [2] have been improved in [6], where also the ellipticity condition could be removed. The main tools in the analysis are a new quasi-interpolation technique and a Helmholtz decomposition lemma for the interpolation error. By relating the implicit error indicators developed in [3, 4] to

the explicit techniques used in [2, 6] we could prove that these implicit error estimators provide both an upper and lower bound for the error [5]. These estimates also provide a mathematical justification for the boundary conditions used for the local error equations. In addition, the restriction in [6] to divergence free source terms has been removed.

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