Effect of micro defects on structure failure : a two scale approach

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Key Words: *failure, asymptotic analysis of boundary perturbations, strong discontinuity model, partition of unity.*

ABSTRACT

This work lies in the context of the prediction of the behavior of damage structures till rupture. Rupture is highly correlated to the presence of defects: material defects, geometrical defects or loading perturbations. In this work, we look more precisely to the prediction of the rupture of complex structures suffering from surfacic singular perturbations. We are concerned by the choice of the most suitable Finite Element strategy to capture the structural behavior. The main feature of our work is to propose an approach dealing both with singular surface perturbations and localization zones development by using a coarse description of the geometry : neither the perturbation shape nor a fine representation of the cohesive crack are considered. To cover the whole response of the structure till rupture, we need to take into account two different phases of the behavior :

- in the first phase (linear elastic), the influence of the geometrical perturbation generating stress concentration has to be evaluated,
- the second phase is characterized by the development, from the perturbation, of a localization zone or a crack propagating through the domain.

Our goal is to design a numerical strategy dealing with a coarse discretization of the unperturbed domain and able to perform the analysis of the structural response from the elastic phase to complete failure. To that purpose, we consider two macroscopic models dedicated to each of the two phases of the behavior.

• The influence of the shape and size of the perturbation is evaluated by an asymptotic analysis for a boundary singular perturbation in an elliptic boundary value problem. A strong use of linearity of the Navier equations is made to evaluate the influence of the defect through a superposition technique [1]. The problem to be solved naturally involves two scales: the structural scale and the perturbation scale. The solution is then approximated by the superposition of the solution of

the unperturbed problem at the structural scale and of a linear combination of profiles solving Navier equations at the scale of the pertubation. We propose a FE strategy based on a kinematic enrichment of standard finite element techniques through the partition of unity [2,3]. The profiles computed from the asymptotic analysis are added to the discrete variational space so that we can reach a fine description of the stress state around the perturbation while using a coarse mesh adapted to the unperturbed domain.

• the description of the localization zones development is provided by the use of a strong discontinuity approach [4,5,6]. Strain localization zones are then taken into account through the introduction of displacement discontinuity surfaces without any need of a fine description of the localization zone. The physical features of the failure process are accounted for by considering adequate discrete type models linking the traction on the discontinuity to the displacement jump. Numerically, such discontinuities are taken into account by enhancing the shape function basis by an Heaviside function through the incompatible mode methods [7].

The key point of the presented approach is to propose a field transfer strategy allowing to couple the enrichment deduced from the asymptotic analysis and the strong discontinuity method. The discrete variational spaces being different we propose a strategy based on energy criteria to project the field from one discrete space to the other one.

We present some numerical results dealing with the failure process of a structure submitted to the influence of one single boundary perturbation. The effect of the size and shape of the perturbation is studied. Moreover, we present some mathematical and numerical results dealing with the failure process of a structure submitted to several perturbations of different sizes and shapes [8]. Finally, we propose some extension to the case of a singular perturbation on a curved boundary.

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