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A projection method and solvers for incompressible viscous flow with Coriolis force

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ABSTRACT

In many physical and industrial applications there is the necessity of numerical simulations for models with moving geometries. In the literature one can find several techniques for handling such type of problems. Among them are fictitious domain, fictitious or immerse boundary methods. Although being quite popular these methods may have accuracy and/or complexity issues when used for simulating complex geometries or high-speed flows. At the same time there is a large class of "rotating" models, when the application of the above methods can be naturally avoided by considering a static computational domain in a rotating reference frame. In the new reference frame the system of Navier-Stokes equations for viscous incompressible fluid takes the form:

$$\begin{array}{rcl} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nu\Delta\boldsymbol{u} + \boldsymbol{\omega}\times\boldsymbol{u} + \nabla p &=& \boldsymbol{f} \\ \nabla\cdot\boldsymbol{u} &=& 0 \end{array} \quad \text{in } \Omega\times(0,T]\,, \tag{1}$$

where $\frac{1}{2}\boldsymbol{\omega}$ is the angular velocity vector, $\boldsymbol{f}$ includes centrifugal forces and $\boldsymbol{\omega}\times\boldsymbol{u}$ are the Coriolis forces. Some boundary or initial conditions should be added to (1).

A popular technique for the time integration of (1) is so-called projection method. Actually the is a variety of splitting schemes which are known in the literature by this name [1]. For the sake of clarity we consider a projection method which has the first order accuracy in time and resembles the well-known Chorin scheme. Higher order methods can be derived in the standard way. The new feature of our method is that the projection and pressure correction steps are modified to account for Coriolis terms. The pressure correction q satisfies the elliptic equation which can be written in the mixed form as

$$\begin{array}{rcl} \alpha(\boldsymbol{x})\boldsymbol{v} + \boldsymbol{\omega}\times\boldsymbol{v} + \nabla q &=& \boldsymbol{0} \\ \nabla\cdot\boldsymbol{v} &=& g \end{array} \quad \text{in } \Omega, \tag{2}$$

where $\alpha(\boldsymbol{x}) \simeq \frac{1}{\Delta t}$ for time step Δt and possibly depends on the velocity, $\boldsymbol{v}$ is an auxiliary vector-function. Some boundary conditions should be supplied in (2). However, in practice we consider the discrete variant of the projection method dealing with matrices, which result after finite element space

discretization, rather than with differential operators. This allows us to avoid the issue of boundary conditions for pressure correction q. We show that (2) leads to the system of algebraic equations for the discrete pressure update with *symmetric sparse* matrix, which can be efficiently solved by conventional methods such as multigrids.

Setting $\alpha(\boldsymbol{x}) = \frac{1}{\Delta t}$ and $\boldsymbol{w} = 0$ in (2) gives the usual projection method of Chorin's type. For several model and more realistic problems we will demonstrate numerically that the new method leads to more stable and time efficient calculations, especially for the case when the angular velocity is not too small. Theoretical considerations show that the methodology is closely related to building a preconditioner for the Schur complement of linearized Navier-Stokes system, the issue extensively studied in the literature over the last decade. Finally we discuss efficient multigrid methods to solve the auxiliary velocity and pressure subproblems on every time step.

The presentation is partially based on the recent papers [2] and [3].

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