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An unsplit variational perfectly matched layer technique optimized at grazing incidence: application to the spectral-element method

*** R. Martin[1,2], D. Komatitsch[1,2,3] and N. Legoff[2]**

[1] CNRS, Laboratoire de Modélisation et Imagerie en Géosciences de Pau
Université de Pau, France
roland.martin@univ-pau.fr, web.univ-pau.fr/ rmartin1, dkomati1

[2] INRIA, Équipe-projet Magique3D
Avenue de l'Université, BP 1155, France

[3] Institut universitaire de France, 103, boulevard Saint-Michel, 75005 Paris, France
dimitri.komatitsch@univ-pau.fr

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ABSTRACT

In the context of wave propagation simulation, the perfectly matched layer (PML) absorbing boundary condition has proven to be efficient to absorb non grazing incidence waves and surface waves, but the classical discrete PML becomes significantly less efficient at grazing incidence. This is a severe limitation in the case of thin mesh slices or in the case of sources located close to the absorbing boundaries or receivers located at large offset. In order to improve the efficiency of PML at grazing incidence, in Komatitsch and Martin (2007) we derived an unsplit convolutional technique (called CPML) for the staggered-grid finite-difference integration scheme. Here we extend this CPML method to a variational version of the seismic wave equation, focussing in particular on the spectral-element method. A hybrid first/second-order time integration scheme is introduced. Using the Newmark time marching scheme, Festa and Vilotte (2005) have shown that a velocity-stress formulation in the PML and a second-order displacement formulation in the inner computational domain match perfectly at the entrance of the CPML. The main difference between our unsplit CPML and the split formulation of the GFPML of Festa and Vilotte (2005) lies in the fact that memory storage is reduced by 40% in 2D in the CPML version. In both CPML and GFPML so-called memory variables are involved and are added to the velocity and stress fields. In 2D, thirteen arrays (2 components of the velocity, 3 components of the stress tensor, 8 memory variables) are involved for the CPML compared to 20 arrays for the GFPML. Furthermore the CPML formulation is easier to implement. We show benchmarks on thin slices for a two-layer model in the presence of a free surface.

The snapshots of Figure 1 show that waves can be efficiently absorbed in a heterogeneous isotropic model in the presence of a free surface. In Figure 2, comparisons with a reference solution illustrate the high efficiency of the variational CPML even at grazing incidence, since only very small discrepancies are observed. In this same figure, total energy exhibits an exponential decay in time over several orders of magnitude, which shows the stability of the variational CPML for long simulation times.

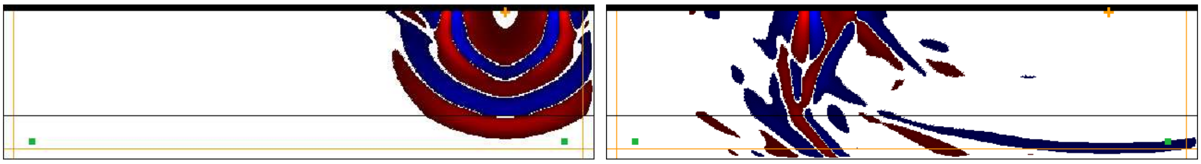

Figure 1: Snapshots of the propagation of P-SV waves excited by a Ricker wavelet source located at the orange cross in ($x = 8500$ m,$y = 2500$ m) at the free surface of a 10000 m by 2500 m domain. Two different materials are in contact at the discontinuity represented by the solid black line. CPML layers are implemented on the right, bottom and left edges. Snapshots are shown at times 0.7 s and 2.8 s. No significant spurious reflections can be observed, even at grazing incidence.

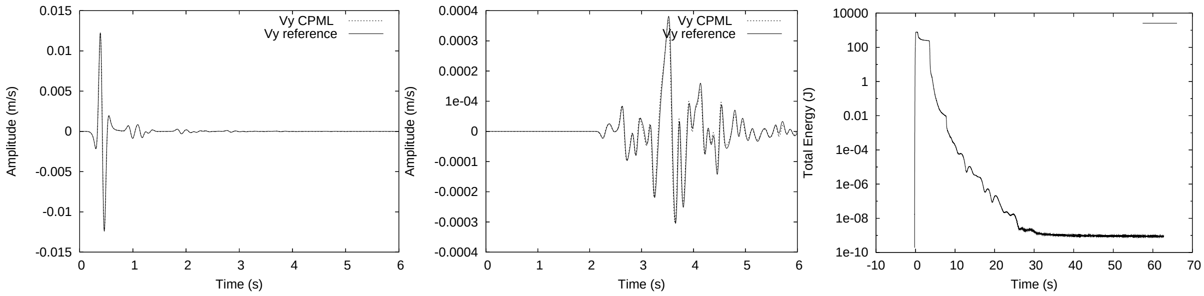

Figure 2: (Left and middle): Time evolution of the numerical solution with CPML (dotted line) for the vertical component of velocity recorded at the first receiver ($x = 9500$ m, $y = 300$ m, green square at the bottom right of the snapshots) and at the second receiver located in ($x = 500$ m, $y = 300$ m, green square located at the bottom left) compared to the exact reference solution (solid line). At these receivers located close to the CPML layer (15 grid points away from its beginning) the agreement is good in spite of the grazing incidence and no spurious oscillations can be observed. (Right): total energy for a long (62 s) simulation decreases continuously from a maximum value of 792 J to a minimum value of $8.76\,10^{-10}$ J. No instabilities are observed on this semi-logarithmic curve, which means that the discrete CPML is stable up to 36000 steps. On the right, one can notice tiny oscillations owing to the fact that total energy is so small that we start to see the effect of roundoff of floating-point numbers of the computer.

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