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Adaptive Multiscale Methods for Elliptic Problems

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ABSTRACT

One of the most challenging problem in computational mechanics is numerical solution of multiscale phenomena, that is, phenomena that are governed by physical processes occurring on a wide range of time and/or length scales. These methods are usually refered to as multiscale methods and are characterized by that they incorporate fine-scale information into a set of coarse-scale equations in a way that is (hopefully) more efficient than solving the fine-scale problem directly. The error introduced by not solving the fine-scale problem may from the numerical coarse-scale problem be looked upon as modeling error, which is to be added to the approximation error in the numerical solution of the coarse scale problem. Herein, we aim for controlling both the approximation error and the modeling error for elliptic multiscale problems.

In many applications within civil, mechanical, naval and petroleum engineering the primary aim of a finite element analysis is to recover forces, stresses or flow fluxes with good accuracy in certain regions of the computational domain. However, in other cases one may aim for solving the problem at hand with good overall accuracy (measured in energy norm) throughout the domain. In any case we always have one or more identified quantities we want to compute within a certain error tolerance which we denote as the quantity/quantities of interest. Goal oriented procedures for computing the quantities of interest consist of three parts; the computation of the requested quantity, an estimate of the error in this quantity, and the computation of element wise refinement indicators which suggest where the model should be refined. Different research groups have addressed goal oriented adaptivity, and we will here pursue the approach introduced by Kvamsdal (1998) (see also (Melbø and Kvamsdal, 2003)).

We will address challenges in geomechanics related to exploration of oil fields. The primary cause of multiscale behavior is heterogeneities in rock and sand formations, occurring at all scales from the pore scale to the scale of the entire model. These heterogeneous structures are reflected in the coefficients of the governing partial differential equations. For solid mechanics applications the coefficients are the Young modulus and the Poisson ratio, whereas for porous media flow this typically is the rock permeability.

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