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Integral equation approaches for estimating gas-damping in MEMS in the free-molecule regime

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ABSTRACT

The specific class of inertial MEMS sensors (e.g. accelerometers, gyroscopes and microphones) is now experiencing an increasing commercial success but still poses several scientific challenges. These MEMS typically consist of collections of fixed parts and vibrating shuttles separated by gaps which are few microns wide. Resonating frequencies are in the order of few kHz and the shuttle velocity is always small with respect to the mean molecular velocity.

The range of working pressures is extremely large, starting from standard conditions down to almost vacuum. There exist a pressure threshold, which depends on the combination of layout, materials and processes employed, above which gas-structure interaction becomes the dominant source of dissipation and intrinsic, solid dissipation can be neglected. This threshold typically occurs between 10^{-5} and 10^{-4}bar. This dissipation strongly affects the quality factor Q which is a key factor in quantifying the performance of the MEMS in industrial environments.

Reliable tools exist for the prediction of gas damping at standard conditions but the situation is less defined at lower pressures when rarefaction effects starts appearing. These are conventionally characterized by the Knudsen number, defined as $\mathrm{Kn} = \lambda/\ell$, where ℓ is a characteristic length of the flow and λ the mean free path of gas molecules. Based on the Knudsen number, gas flows can be qualitatively classified as continuum ($\mathrm{Kn} < 0.01$), slip ($0.01 < \mathrm{Kn} < 0.1$), transition ($0.1 < \mathrm{Kn} < 10$), and free-molecule ($\mathrm{Kn} > 10$).

The stochastic simulation method known as the direct-simulation Monte Carlo (DSMC) has been developed to circumvent this difficulty in the transition regime and has been very successful in simulating flows with high Kn and Mach numbers. Several limitations, however, still hinder its application to more realistic situations. A recent trend emerging in this area is to adopt a deterministic approach for the solution of kinetic equations which becomes viable if the complicated collision integral in the BE is replaced by simpler expressions. However its application to realistic 3D MEMS is still under investigation.

On the contrary, the collisionless or free-molecule flow lends itself to the development of simpler numerical models which are relatively un-investigated and tested. Based on the the formal classification given above, the free-molecule flow represent the limiting case where the Knudsen number tends to infinity. In typical inertial MEMS this applies typically at pressures in the range of 1 mbar and below. Hence the pressure range where this simple model applies is always meaningful. Moreover starting

from the quantitative evaluation of dissipation in the free-molecule and slip-flow regime, simple bridging formula could be applied to obtain at least a rough estimate of the dissipation in the transition regime.

Hence, in the following we focus on the assessment of two different techniques for the analysis of free-molecule flows with some simplifying assumptions discussed below. It will also be assumed that the shuttle movement can be expressed as: $\mathbf{s}(\mathbf{x},t) = \Psi(\mathbf{x})q(t)$ where $\Psi(\mathbf{x})$ is a function to be specified and in most cases corresponds to a modal shape of shuttle and springs. This implies that a sort of decoupling is assumed between the structural response and the fluid action, the latter never affecting the shape of the assumed displacement field but only the weight $q(t)$. Moreover the hypothesis that the shuttle velocity is small with respect to thermal molecular speeds leads to the formulation of linear models. As a consequence, the forces exerted by the fluid on the structure have the simplified form $\mathbf{p}(\mathbf{x},t) = \mathbf{t}(\mathbf{x})\dot{q}(t)$, where $\mathbf{t}(\mathbf{x})$ is a complex function in the general case and reduces to a real function of $\mathbf{x}$ for the low resonating frequencies of interest herein. Employing classical structural theories for flexible beams, an application of the principle of virtual work allows to reduce the global analysis to the 1D model $M\ddot{q}(t) + B\dot{q}(t) + Kq(t) = F(t)$ where, if S_{SH} denotes the surface of the shuttle, the damping coefficient B is:

$$B = \int_{S_{\mathrm{SH}}} \Psi(\mathbf{x})\mathbf{t}(\mathbf{x})\mathrm{d}S \tag{1}$$

In the case of forced sinusoidal excitation at resonance the solution will have the form $q(t) = A\sin(\omega_0 t)$ and it is customary to define the quality factor of the structure as the ratio $Q = 2\pi E/D$ where E is the maximum elastic energy stored in the system and D is the dissipation per unit-time $D = \pi\mathrm{Re}(B)A^2\omega_0$.

In general, the numerical technique employed for estimating the quality factor of MEMS at low pressure is Molecular Dynamics (MD). Its basic concepts will be briefly reviewed. However, this approach is stochastic in nature which is a major drawback for the extremely low-speed applications at hand. A different technique based on integral equations seems very competitive with respect to MD for working conditions of inertial MEMS since it is deterministic, fast and robust as discussed herein after reviewing the underlying key assumptions of kinetic theory. Finally several examples are analysed including a benchmark with experimental data.

REFERENCES

[1] Cercignani C. “The Boltzmann equation and its applications”. Springer Verlag, New York, 1988.

[2] Frangi A, Spinola G., Vigna B. “On the evaluation of damping in MEMS in the slip-flow regime”. *Int. J. Numer. Methods Engng.*, Vol. **68**, 1031-1051, 2006.

[3] Liu Y.J., Shen L. “A dual BIE approach for large-scale modelling of 3-D electrostatic problems with the fast multipole boundary element method”. *Int. J. Numer. Methods Engng.*, Vol. **71**, 837-855, 2007.