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STRESS INTENSITY FACTORS ANALYSES OF AN THREE-DIMENTIONAL INTERFACIAL CORNER BETWEEN DISSIMILAR ANISOTROPIC MATERIALS UNDER THERMAL STRESS

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ABSTRACT

Micro-structures such as those utilized in electronic devices and micro-electro mechanical systems (MEMS) are composed of many different materials. Many interfacial corners exist in electronic devices and MEMS because each of the materials employed has a different configuration. Due to the mismatch of the materials' thermal expansion and elastic properties, the stress concentration at an interfacial corner causes failure. Therefore, the strength of an interfacial corner is very important for the reliability of an electronic product.

According to the theory of linear elasticity, asymptotic stress near the tip of a sharp interfacial corner is generally singular as a result of a mismatch of the materials' elastic constants. The singular order and the eigenfunctions are obtained using the Williams eigenfunction method, which depends on the materials' properties and the geometry of an interfacial corner. The asymptotic solutions near the tip of a corner under thermal stress have been expressed [1].

$$\begin{aligned} \sigma_{ij}^{k} &= \sum_{m=1}^{\mathrm{N}} C_m r^{\lambda_m - 1} f_{ij}^{mk}(\theta) + \sigma_{ij0}^{k}(\theta) \\ u_i^{k} &= \sum_{m=1}^{\mathrm{N}} C_m r^{\lambda_m} g_i^{mk}(\theta) + u_{i0}^{k}(r,\theta) \end{aligned} , \qquad (1)$$

where (r, θ) is the polar coordinate whose origin is located on the corner tip. C_m (m=I,II, … N) is a scalar coefficient obtained by the H-integral, λ_m is the singular order and N is the number of singular orders.

A numerical method using the path-independent H-integral based on the Betti reciprocal principle was developed to analyze the scalar parameters of an interfacial corner between anisotropic bimaterials by Labossiere [2]. We extended this integral to the thermal anisotropic elastic problem using the body force analogy [3].

$$H = \int_{C_r} (\sigma_{ij} u_i^* - \sigma_{ij}^* u_i) n_j ds + \int_{\Omega} \beta_{ij} \vartheta \varepsilon_{ij}^* d\Omega , \qquad (2)$$

where σ_{ij} and u_i are actual stress and displacement, respectively. σ_{ij}^*, u_i^* and ε_{ij}^* are

complementary stress, displacement and strain, respectively, which satisfy the same equilibrium and constitutive relations as the actual fields. The integral path C_r is arbitrary from the lower flank to the upper flank. ϑ is the change of temperature. β_{ij} is defined as

$$\beta_{ij} = C_{ijks}\alpha_{ks}, \tag{3}$$

where C_{ijks} is the stiffness and α_{ks} is the coefficient of thermal expansion. The stress and displacement around an interfacial corner for the H-integral are obtained using the finite element method. If appropriate complementary stress and displacement are selected, the scalar coefficient C_m can be obtained by the H-integral. The stress and displacement around an interfacial corner for the H-integral are obtained using the finite element analysis. In the case of three-dimensional problem, Eq. (2) is modified as

$$H = \int_{C_r} (\sigma_{ij}u_i^* - \sigma_{ij}^* u_i) n_j ds + \int_\Omega (\sigma_{i3,3}u_i^* - \sigma_{i3}^* u_{i,3}) d\Omega + \int_\Omega \alpha_{ij}\vartheta\sigma_{ij}^* d\Omega . \tag{4}$$

A new definition of the stress intensity factors of an interfacial corner was proposed as

$$\mathbf{k} = \begin{Bmatrix} K_{\mathrm{II}} \\ K_{\mathrm{I}} \\ K_{\mathrm{III}} \end{Bmatrix} = \lim_{\substack{r\to 0\\ \theta=0}} \sqrt{2\pi}\, l_k^{1-\mathrm{Re}[\lambda_1]} \mathbf{\Lambda}(\theta) \left\langle (r/l_k)^{1-\lambda_m} \right\rangle \mathbf{\Lambda}^{-1}(\theta) \begin{Bmatrix} \sigma_{12} \\ \sigma_{22} \\ \sigma_{32} \end{Bmatrix}, \tag{5}$$

$$\mathbf{\Lambda}(\theta) = \left[\mathbf{F}^{\mathrm{I}}(\theta) \quad \mathbf{F}^{\mathrm{II}}(\theta) \quad \mathbf{F}^{\mathrm{III}}(\theta) \right], \tag{6}$$

where l_k is the characteristic length, and $\mathbf{F}^{\mathrm{I}}(\theta)$, $\mathbf{F}^{\mathrm{II}}(\theta)$ and $\mathbf{F}^{\mathrm{III}}(\theta)$ are eigen functions. The relation between the scalar coefficient C_m and these stress intensity factors is

$$\begin{Bmatrix} K_{\mathrm{II}} \\ K_{\mathrm{I}} \\ K_{\mathrm{III}} \end{Bmatrix} = \sqrt{2\pi} [C_{\mathrm{I}} l_k^{\mathrm{Im}[\lambda_1]} \begin{Bmatrix} f_{12}^{\mathrm{I}}(0) \\ f_{22}^{\mathrm{I}}(0) \\ f_{32}^{\mathrm{I}}(0) \end{Bmatrix} + C_{\mathrm{II}} l_k^{\lambda_{\mathrm{II}} - \mathrm{Re}[\lambda_1]} \begin{Bmatrix} f_{12}^{\mathrm{II}}(0) \\ f_{22}^{\mathrm{II}}(0) \\ f_{32}^{\mathrm{II}}(0) \end{Bmatrix} + C_{\mathrm{III}} l_k^{\lambda_{\mathrm{III}} - \mathrm{Re}[\lambda_1]} \begin{Bmatrix} f_{12}^{\mathrm{III}}(0) \\ f_{22}^{\mathrm{III}}(0) \\ f_{32}^{\mathrm{III}}(0) \end{Bmatrix}]. \tag{7}$$

The asymptotic solutions of stress and displacement around an interfacial corner are uniquely obtained using these stress intensity factors. These stress intensity factors are directly connected to those of interfacial cracks proposed by Hwu [4] and of homogeneous cracks.

The stress intensity factors of a three-dimensional interfacial corner were obtained accurately using H-integral for three-dimensional problems as shown in Eq. (4).

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