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The Rayleigh-Benard problem in parallelepiped enclosures with Soret effect

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ABSTRACT

The stability of liquid layers heated from below, the classical Rayleigh-Benard problem, has been extensively analyzed in the literature (see [3]and the reference therein). It is also widely studied, the case when the liquid is a binary mixture, so when the Soret effect has to be taken into account, see [6]. However, less is known when the fluid layer is confined with lateral boundaries. In the present paper, following work [6], we analyze the stability of a binary mixture when this fluid is confined between four rigid and adiabatic side walls, and a perfectly conducting top and bottom walls.

The linearizated PDE set governing the evolution of the system is taken from [6], the unknowns are the stream function $\psi(x, y, z)$, related with fluid velocity field by $v_x = 0$, $v_y = \partial_z \psi$, and $v_z = -\partial_y \psi$, the temperature perturbation field $T(x, y, z)$ and $\zeta(x, y, z) = C(x, y, z) + T(x, y, z)$ where $C(x, y, z)$ is the mass function field.

The zero velocity field on the six faces implies that $\psi(x, y, z)$ must satisfy Cauchy homogeneous conditions on $y = \pm 1$ and $z = \pm 1$, and Dirichlet homogeneous condition on $x = \pm 1$. As the temperature is prescribed at the bottom and top boundaries, Dirichlet homogeneous conditions for T must be satisfied on $z = \pm 1$, and no heat flux across four lateral boundaries implies Neumann homogeneous conditions for T on $y = \pm 1$ and $z = \pm 1$. Finally Neumann homogeneous boundary conditions on $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$ for ζ have to be taken into account.

The governing equations and boundary conditions constitute an eigenvalue problem for the Rayleigh number $\left(Ra = \frac{g\beta_T \Delta T H^3}{\nu\alpha}\right)$ where α is the thermal diffusivity. In order to solve the characteristic eigenvalue equations for arbitrary aspect ratios, A, B, separation ratio S and Lewis number $Le = \alpha/D$, where D is the molecular diffusion, Galerkin spectral techniques and collocation pseudospectral techniques are considered.

When Galerkin is applied, the dependent variables can be expanded in triple-truncated series of trial functions[1] that satisfy above boundary conditions. Two kinds of trial functions families are taken into account, Fröbenius type functions, like those used in [5] or [6], and Chebyshev polynomials.

[1] see for instance [1]

For collocation method only Chebyshev expansions is used. However, two implementation algorithms are developed, one as done in [4] using the same trial functions as those used in the Galerkin method, and the other using a matrix differentiation method[2] (MMT).

Some of the obtained results are shown in table 1 and in figure 1.

Method:	Galerkin		Collocation
N=M=K	*Fröbenius*	*Chebyshev*	*Chebyshev*
1	253.9406	637.4271	74.6178
2	209.3110	209.1281	208.6987
3	208.9085	209.0300	206.5871
4	208.8936		208.6014
5	208.8920		208.8072

Table 1: Results of critical Rayleigh (R_c) number with $A = B = 1$, $M = N = K$ and $Le = 100$. Galerkin method using chebyshev functions takes to long.

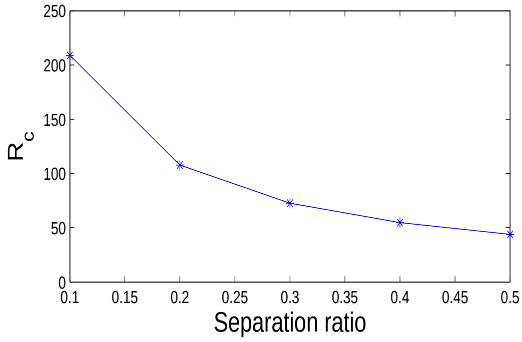

Figure 1: Evolution of Ra_c versus separation ratio when $A = B = 1$ and $Le = 100$

In conclusion, from a physical point of view we can obtain the dependence between the critical Rayleigh number Ra_c, and Lewis number, separation ratio and aspect ratios, as it is shown in figure 1.

On the other hand, spectral methods with Fröbenius type trial functions are the most accurate method for this physical problem, so is the best method when short expansion of the unknowns are considered. However, collocation method runs faster than the spectral method, and this behaviour is more accentuated when MMT algorithm is applied, therefore, we are allowed to obtain better results for the same spent time of CPU. Nevertheless, impose boundary conditions is harder to introduce in MMT using algorithms.

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[2] see for instance [2] or [7]