8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

SPATIAL STABILITY OF LINEAR MULTISTEP METHODS FOR FIRST-ORDER TRANSIENT EQUATIONS

*** Isaac Harari[1] and Eran Grosu[2]**

Faculty of Engineering, Tel Aviv University
69978 Ramat Aviv, Israel
[1] harari@eng.tau.ac.il, www.eng.tau.ac.il/~harari
[2] grosu@eng.tau.ac.il

Key Words: *Semidiscrete Formulations, Small Time Step Oscillation, Rothe Method, Spatial Stabilization.*

ABSTRACT

Solutions of direct time integration schemes that converge in time to conventional semidiscrete formulations may be polluted at small time steps by spurious spatial oscillations, along with attendant overshoot in time. These pathologies are the deleterious effects of higher modes of spatially discrete formulations, which are approximated poorly, and are inevitably admitted into the computation as the time step is reduced with a fixed mesh. This degradation is not an artifact of the time-marching scheme, but rather a property of the solution of the semidiscrete formulation itself.

An analogy to singularly perturbed problems by the Rothe method (or horizontal method of lines), of discretizing in time and then in space on each discrete time level, provides an upper bound on the time step for the onset of spatial instability. Previous work applied this concept to one-step methods for diffusion problems [1], advection-diffusion-reaction [2], and elastodynamics [3]. The analogy is extended to derive a simple procedure of spatial stabilization that removes the pathology, leading to stabilized implicit time-integration schemes that are free of spurious oscillations at small time steps

The present work extends these ideas to linear multistep methods [4]. Implicit algorithms included in the widely-used LSODE [5] and VODE [6] software packages such as Adams-Moulton and BDF methods are considered.

REFERENCES

[1] I. Harari. "Stability of semidiscrete formulations for parabolic problems at small time steps." *Comput. Methods Appl. Mech. Engrg.*, Vol. **193**, 1491–1516, 2004.

[2] I. Harari and G. Hauke. "Semidiscrete formulations for transient transport at small time steps." *Internat. J. Numer. Methods Fluids*, Vol. **54**, 731–743, 2007.

[3] E. Grosu and I. Harari. "Stability of semidiscrete formulations for elastodynamics at small time steps." *Finite Elem. Anal. Des.*, Vol. **43**, 533–542, 2007.

[4] C.W. Gear. *Numerical Initial Value Problems in Ordinary Differential Equations*. Prentice-Hall, 1971.

[5] A.C. Hindmarsh. "ODEPACK, a systematized collection of ODE solvers." pp. 55–64 in *Scientific Computing*, R.S. Stepleman et al., eds. North-Holland, 1983.

[6] P.N. Brown, G.D. Byrne, and A.C. Hindmarsh. "VODE, a variable-coefficient ODE solver." *SIAM J. Sci. Stat. Comput.*, Vol. **10**, 1038–1051, 1989.