8th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

# A Three-Dimensional Model of Cellular Electrical Activity

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**Key Words:** *electrophysiology, interface problems, implicit schemes, finite volume method*

## ABSTRACT

Cellular electrical activity is central to cellular physiology, and it has been one of the most successful examples of mathematical modeling in biology. Most models of cellular electrical activity are based on the cable model, in which an ohmic current continuity relation results in a one-dimensional reaction diffusion system.

In the derivation of the cable model, one assumes that the ionic concentrations do not change appreciably over the time of interest, and that a one-dimensional picture of cell geometry is adequate for purposes of describing cellular electrical activity. We present a three-dimensional model of cellular electrical activity that takes into account both ionic concentration and geometrical effects on electrophysiology. We view biological tissue as three-dimensional space partitioned by the cell membrane into intracellular and extracellular spaces. The system of partial differential equations consists of drift-diffusion equations for each ionic species supplemented with the electroneutrality condition. Boundary conditions satisfied at both sides of the cell membrane model the capacitative nature of the membrane as well as the ionic channel currents. The model may be described mathematically as a system of partial differential algebraic equations with nonlinear evolutionary interface conditions. This system has the virtue of being more general in its physiological applicability than the cable model, but has the difficulty of being far more complicated to study either analytically or numerically.

We present a numerical method for solving the above equations numerically A challenge in constructing a numerical scheme for this model is that its equations are stiff: There is a time scale associated with 'diffusion' of the membrane potential that is much faster than the time scale associated with the physical diffusion of ions. We use an implicit discretization in time and a finite volume discretization in space. We present convergence studies of the numerical method for several cases of physiological interest.

## REFERENCES

[1] Y. Mori, J.W. Jerome and C.S. Peskin *A Three-Dimensional Model of Cellular Electrical Activity*, Bulletin of the Institute of Mathematics, Academia Sinica, Volume **2**, No.2. pp. 367-390, Jun. 2007.

[2] Y. Mori and C.S. Peskin *A Numerical Method for Cellular Electrophysiology based on the Electrodiffusion Equations with Internal Boundary Conditions at the Membrane*, submitted.