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A Modified Block Element Method

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ABSTRACT

Block Element Method[1] is a discontinuous method for the analysis of fissured rock mass and multi-interface structure. Generally, the deformation of the block is ignored or only the elastic deformation of it is considered in the method with rigid body displacement model. In this paper a modified block element method presents (MBEM), in which the constant strain of the block is involved and the basic unknown quantities are the rigid displacements and the constant strains of the block elements.

It is assumed that the number of the block elements (V1 in Figure 1)and of the joint elements (V2 in Figure 1) are N_B and N_C separately in the calculation domain. The basic unknown quantities in the element i are:

$$\boldsymbol{d}_i^e = \begin{Bmatrix} \boldsymbol{\delta}_i^e \\ \boldsymbol{\varepsilon}_i^e \end{Bmatrix} \tag{1}$$

$\boldsymbol{\delta}_i^e = [u_i \quad v_i \quad w_i \quad \theta_{xi} \quad \theta_{yi} \quad \theta_{zi}]^T$ and $\boldsymbol{\varepsilon}_i^e = [\varepsilon_{xi} \quad \varepsilon_{yi} \quad \varepsilon_{zi} \quad \gamma_{xyi} \quad \gamma_{yzi} \quad \gamma_{zxi}]^T$ are the rigid displacement of the centre and the constant strain of the block element separately.

The governing equation obtained from variational principle is:

$$\boldsymbol{Kd} = \boldsymbol{R} \tag{2}$$

In which $\boldsymbol{d}$ is the global nodel displacement vector, $\boldsymbol{R}$ is the global nodal load vector, and $\boldsymbol{K}$ is the global stiffness matrix defined as below:

$$\boldsymbol{K} = \sum_{m=1}^{N_c} (\boldsymbol{c}_m^e)^T \boldsymbol{k}_m \boldsymbol{c}_m^e + \sum_{i=1}^{N_B} (\boldsymbol{c}_i^e)^T \overline{\boldsymbol{D}}_i \boldsymbol{c}_i^e \tag{3}$$

where $\boldsymbol{K}_m$ is from the joint elements while $\overline{\boldsymbol{D}}_i$ is from the block elements:

$$\boldsymbol{k}_m = \frac{1}{h^2} \int_{v_2} \boldsymbol{N}^T \boldsymbol{L}^T \boldsymbol{D}' \boldsymbol{L} \boldsymbol{N} dv \qquad \overline{\boldsymbol{D}}_i = \int_{v_1} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix} dv \tag{4}$$

In which h is the thickness of joint element between block i and block j(Figure 2), $\boldsymbol{D'}$ and $\boldsymbol{D}$ are the stress-

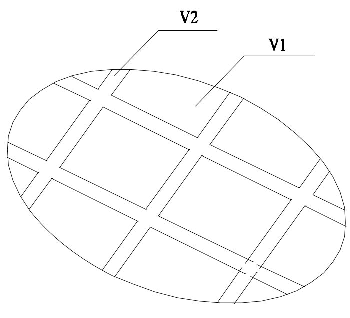

Figure 1 elements of block and joints

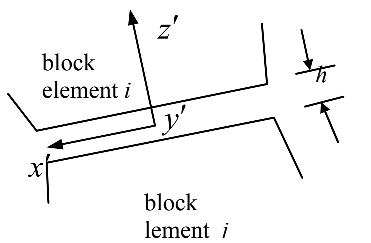

Figure 2 local coordinate in joints

strain matrixes of the materials of the block elements and the joint element separately, ***L*** is the direction cosine matrix of the local coordinates on the joint and the global coordinates, and ***N*** is the shape function matrixes written as:

$$\boldsymbol{N} = [\boldsymbol{N}_i \quad -\boldsymbol{N}_j], \quad \boldsymbol{N}_i = [\boldsymbol{N}_i^{'} \quad \boldsymbol{N}_i^{''}] \tag{5}$$

$$\boldsymbol{N}_i^{'} = \begin{bmatrix} 1 & 0 & 0 & 0 & z-z_{ci} & -y+y_{ci} \\ 0 & 1 & 0 & -z+z_{ci} & 0 & x-x_{ci} \\ 0 & 0 & 1 & y-y_{ci} & -x+x_{ci} & 0 \end{bmatrix} \tag{6}$$

$$\boldsymbol{N}_i^{''} = \begin{bmatrix} x-x_{ci} & 0 & 0 & (y-y_{ci})/2 & 0 & (z-z_{ci})/2 \\ 0 & y-y_{ci} & 0 & (x-x_{ci})/2 & (z-z_{ci})/2 & 0 \\ 0 & 0 & z-z_{ci} & 0 & (y-y_{ci})/2 & (x-x_{ci})/2 \end{bmatrix} \tag{7}$$

As the material enters the plastic state, the effect of plasticity on the constitutive equation should be considered. In the model of MBEM the plasticity and failure of the block can be considered besides the joint. By establishing corresponding constitutive relationships, for examples elastic-brittle, rheological etc, other nonlinear problems can be easily solved by using MBEM.

The example of the simply supported beam indicates that displacement precision of MBEM which considers the deformation of the block elements is much higher than that of Block Element Method with rigid body displacement model. The displacement at the mid-span of the beam and the stresses σ_x are more close to the theoretical solution. It shows the results of MBEM is more approach to practical situation since the structural stiffness becomes soft by increasing the degrees of freedom in MBEM. Further analysis indicates that the results of the Block Element Method with rigid body displacement model is sensitive to the mesh shape but the sensitivity will decrease for MBEM. Therefore, MBEM should be adopted in the analysis of the rock engineering with complex geological structures in which the element shape is usually irregular.

Figure 1 x=0.0m Comparison of the results

	Displacement at the mid-span (cm)	Normal stress of the Mid-span Cross-section σ_x (N/m^2)							
Position y (m)		-0.175	-0.125	0.075	0.025	-0.025	-0.075	-0.125	-0.175
Elasticity[2]	0.2405	576.7	411.9	247.1	82.4	-82.4	-247.1	-412.0	-576.8
Block Element Method	0.0887	574.8	408.8	244.2	81.1	-81.1	-244.2	-408.8	-574.8
MBEM	0.201	575.2	409.1	244.8	81.1	-81.1	-244.8	-409.1	-575.2

REFERENCES

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[2] Xu Zhilun, *A Concise Course in Elasticity*, 3th Edition, Higher Education Press, 2002