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MPS-BASED UNIFIED ALGORITHM FOR COMPRESSIBLE AND INCOMPRESSIBLE FLOWS

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ABSTRACT

Moving Particle Semi-implicit (MPS) method[1] is a Lagrangian particle method which requires no calculation mesh for incompressible flows. MPS is capable of calculating complex fluid flows such as a breaking wave[2] and droplet breakup[3] involving fluid disruption or coalescence. Fluid structure interaction was also solved by MPS[4]. Smoothed Particle Hydrodynamics (SPH) method[5] is another particle method developed for compressible flows. Some researchers conducted incompressible flow calculations based on SPH method.

There are two major approaches for unified algorithm solving compressible and incompressible flows for Eulerian methods. Hirt[6] introduced limited compressibility to incompressible codes. This approach is only for law mach number flows, therefore shock waves cannot be captured. Another approach based on CIP (Cubic Interpolated Pseudo particle) was suggested by Yabe[7]. Yabe's C-CUP (CIP Combined Unified Procedure) method is derived from compressible flow equation, and can capture shock waves.

Though unified procedures have been suggested by researchers for Eulerian methods[6],[7], there are no study about such algorithm based on a particle method. Consequently, we extend MPS method to solve compressible flows as well as incompressible flows by a similar approach to the C-CUP[7] method. We term it MPS-AS (MPS for All Speed) method.

Governing equations are the mass, momentum, energy conservation and gas state equations. We get an evolution equation of pressure from the energy equation as:

$$\frac{1}{\gamma-1}\frac{Dp}{Dt} = -p\nabla\cdot\vec{u} + \frac{\kappa}{\rho}\nabla^2 T + \Phi \tag{1}$$

where, p is pressure divided by density, and Φ is a dissipation function. The other symbols are used as the ordinary meanings, respectively.

Spatial discretization is the same as the original MPS method. Time evolution consists of two steps: prediction and correction phases. In the first phase, the predicted velocity u^* is calculated explicitly as:

$$\frac{\vec{u}^* - \vec{u}^n}{\Delta t} = -\frac{p^n}{\rho^n}\nabla\rho^n + \nu\nabla^2\vec{u}^n + \vec{g} \tag{2}$$

In the second phase, u^* is corrected as:

$$\frac{\vec{u}^{n+1} - \vec{u}^*}{\Delta t} = -\nabla p^{n+1} \tag{3}$$

$$\frac{p^{n+1}-p^{*}}{\Delta t}=-(\gamma-1)p^{*}\nabla\cdot\vec{u}^{\,n+1} \tag{4}$$

p^{n+1} is calculated implicitly by coupling equations (3) and (4) as:

$$\left(\nabla^{2}-\frac{1}{\Delta t^{2}(\gamma-1)p^{*}}\right)p^{n+1}=\frac{1}{\Delta t}\nabla\cdot\vec{u}^{*}-\frac{1}{\Delta t^{2}(\gamma-1)} \tag{5}$$

We applied our method to two-dimensional shock tube and dam break problems for validating capability of solving compressible and incompressible flows, respectively.

As we can see in figure 1, an expansion wave, contact face and shock wave can be captured. Tough there are some overshoot and vibrations, average positions of waves and values in each regions are similar to theoretical solution. These overshoot and vibrations will be suppressed by using a better artificial viscosity.

In the dam break calculation, MPS-AS method can capture the free surface and manage the interaction between water and the solid wall as in figure 2. Water impinges on the right side wall and then goes up.

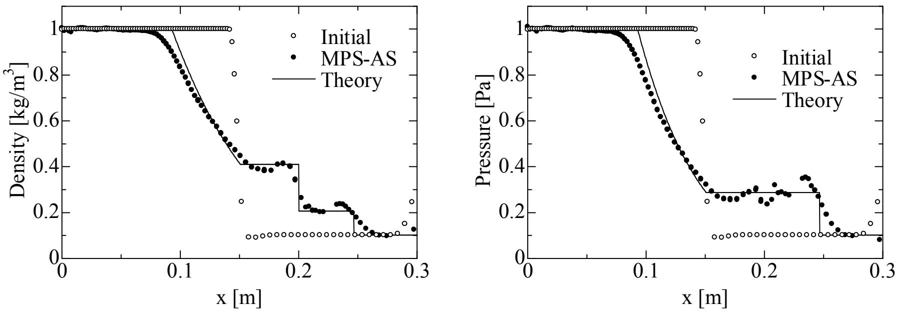

Figure 1. Shock Tube Result

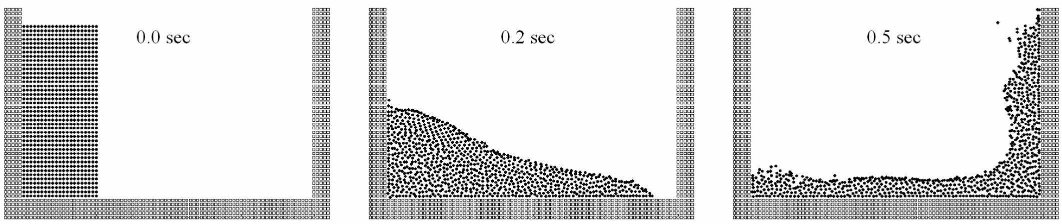

Figure 2. Dam Break Calculation

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