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ON THE COMBINATION OF THE TOPOLOGICAL DERIVATIVE METHOD WITH FREE MATERIAL OPTIMIZATION

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ABSTRACT

One of the basic problems in optimal structural design reads as: find the stiffest elastic structure in a domain Ω, which is subjected to a set of given loads f and fulfills both boundary conditions on the boundary Γ_u and a volume constraint. There are various approaches to solve this problem, one amongst them is Free Material Optimization (FMO) [1]. It uses the entire anisotropic material tensor E at each point $x \in \Omega$ as design variable. To find the stiffest structure the compliance J – the negative potential energy in the equilibrium state – is minimized. Only a limited amount of material can be used for this task resulting in a volume constraint on the material "density" $\operatorname{tr}(E(x))$. Therefore FMO yields not only the optimal material distribution, but also the optimal material properties at each point.

Manufacturers use this information in the predesign phase to find the topology of the optimal structure and to get an idea of the optimal material to use in the production process. Although in the FMO result usually regions with a low material density can be found they are not transferred to the final design. Instead holes are cut out in these areas due to better manufacturability. This immediately leads to the question: where should these holes be drilled and what shape should be used?

While this cannot be answered in the scope of standard Free Material Optimization it is a typical problem for the Topological Derivative method [2]. The Topological Derivative $\mathcal{T}(x)$, $x \in \Omega$, provides information on the infinitesimal variation of the compliance J when a small hole is created at the point $x \in \Omega$. Thereby it is possible to find the optimal spot for drilling a hole. Finding the optimal shape of its boundary is then handled within the limits of shape optimization.

Thus we propose a hybrid method combining Free Material Optimization and the Topological Derivative method. Starting from the solution of a standard FMO problem we detect regions with low "density" $\operatorname{tr}(E)$ using a Watershed algorithm. It interprets the amount of material at a point $x \in \Omega$ as the height in a landscape. The resulting relief is then flooded from below, with holes pierced in local minima. The regions with small density are merged into the set Ω_1, the remainder of the domain $\Omega \setminus \Omega_1$ is denoted by Ω_2.

We now intend to use the Topological Derivative Method in Ω_1. Though it is assumed in the calculation of the Topological Derivative that the material is fixed we cannot continue using E as a variable in Ω_1. For this reason we approximate the material in Ω_1 by an equal amount of isotropic material using an isotropic best approxmation for anisotropic material [3]. The FMO problem with fixed isotropic material in Ω_1 can still be solved using the same methods as in standard FMO. This is done by converting the FMO problem into a nonlinear convex semidefinite program and solving it e.g. by the nonlinear SDP code PENNON [4]. The resulting material distribution and displacement field serve as a starting point for a domain decomposition approach [5], [6]. Thus we separate the isotropic elasticity problem in Ω_1 from the Free Material Optimization problem in Ω_2. These two problems on the subdomains are connected by transmission conditions for the displacements and the stresses on the boundary $\Gamma = \overline{\Omega}_1 \cap \overline{\Omega}_2$. We are now able to compute the adjoint variables for the isotropic elasticity problem in Ω_1 and consequently the topological derivative as given in [7]. Thus we gain information about the optimal location $x^* \in \Omega_1$ for drilling a hole.

Having found the optimal spot x^* for the hole we return to the original problem with anisotropic material in the entire domain Ω. By using a levelset method, as described, for example, in [8], the boundary of the created hole is pushed to its optimal shape. As the modification of the original domain Ω influences the optimal solution of the FMO problem the solution of the Free Material Optimization problem has to be recomputed on the domain with the hole. The resulting change in material on the other hand enters into the expression for the shape derivative. This leads to an alternating computation of the optimal material and the optimal boundary until an equilibrium state optimal for both problems is reached. Afterwards we can again use a Watershed algorithm and apply the Topological Derivative method until the objective cannot be improved by creating holes in the regions with low density. The resulting design will still make use of advanced materials as in standard Free Material Optimization, but it contains holes with sharp edges making it a more realistic design.

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