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Toward modular multigrid design optimisation

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ABSTRACT

Numerical design optimisation based on CFD is still not a routine application due to the high computational cost of the evaluation of the goal function. The most efficient methods use adjoint-based sensitivities [1]. Classically, a fully sequential approach to converging the system has been adopted [5] where at each evaluation for functional and gradient the flow state (primal) and the adjoint (dual) are fully converged. These strict convergence requirements have been shown to be unnecessary for adjoint methods. A number of “simultaneous time-stepping” or “one-shot” methods have been proposed [2-4,7] which converge primal, dual and design simultaneously and compute an optimal design in 5-10 times the cost of the evaluation of the primal and dual. Both algorithms [3,7] have been subsequently extended to use multigrid for primal and dual, but the most promising approach is to include the design iterations in the multigrid algorithm to enhance the convergence rate to the optimal design. This is particularly relevant for realistic industrial applications requiring a large number of design parameters. Industrial application will also require a high level of automation in the definition of the design parameters, one approach is to use an automatically selected subset of the surface points of the CFD mesh as design variables which are then used to morph the volume mesh. Unfortunately this further increases the number of degrees of freedom in the design problem which can lead to oscillatory shapes [5].

Implementation of the multigrid solver requires good high frequency damping properties. This paper presents a comparison of various explicit and implicit smoothing operators for the infinite-dimensional case using all surface points as design variables. Best convergence and accuracy is achieved for point-Jacobi smoothing of the surface displacement, outperforming the commonly used implicit smoothing of the gradients [5].

Another key problem for the development of a multigrid solver for design is the definition of the coarse level functional and the paper presents two approaches. The formulation by Lewis and Nash [8] (EMG) modifies the coarse grid functional to preserve stationarity of the fine level on the coarse grid $L_H = L(u_H, \alpha_H) - L(I_h^H u_h, I_h^H \alpha_H) + I_h^H L_h$. This approach is fully analogous to the standard FMG algorithms for state and adjoint. Alternatively, a simplified formulation (SMG) does not alter the coarse grid functional $L_H = L(u_H, \alpha_H)$ and leaves it to the line-search to safe-guard the convergence.

Numerical experiments were performed for an inverse design case to minimise the square of the pressure difference to an RAE2822 aerofoil starting from a NACA 0012 aerofoil in inviscid flow on two

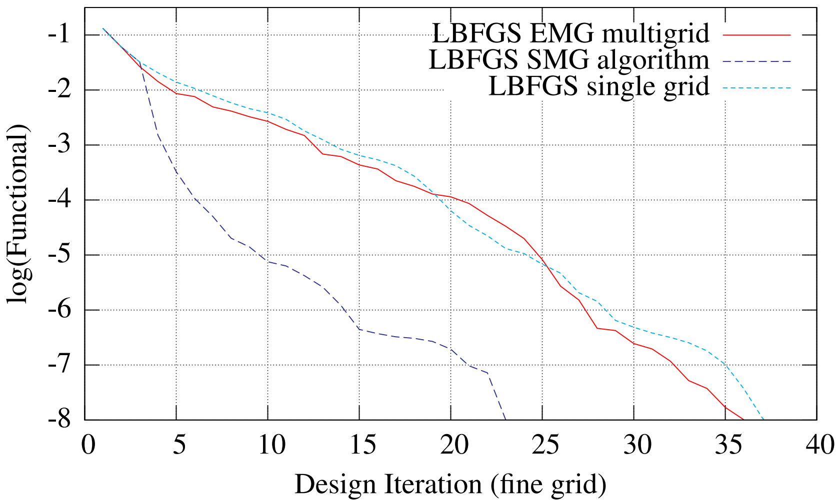

Figure 1: Performance of two multigrid formulations compared to single grid L-BFGS optimisation in fine grid design iterations.

grid levels. Fig. 1 presents the performance of the multigrid algorithm as convergence of the functional vs. the number of design iterations on the fine grid. The functional converges in fewer iterations using the SMG formulation, in particular when seeking "industrial" levels of convergence (10^{-5}). The EMG formulation converges at the same rate as single grid approach which is thought to be linked to insufficient high frequency smoothing of the design variables. The paper will present a detailed analysis of the smoothing behaviour and present results on multilevel grid sequences.

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