

A CLASS OF HIGH-ORDER AND MULTIVARIATE INTERPOLATION METHODS FOR ADAPTING REDUCED-ORDER MODELS TO CONTINUOUS PARAMETER CHANGES

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ABSTRACT

There are several important reasons that can lead to choosing a Reduced-order model (ROM) description of a physical system over a full-order description that could offer a more detailed representation of the physical phenomenon. One of these is that ROMs are usually thought of as enabling near real-time analysis. However, as shown by the authors in previous publications ([1]), building a ROM can be computationally expensive. This issue, as well as the lack of robustness of a given ROM with respect to physical or modeling parameter changes led to the development by the authors ([2]) of a differential geometry-based method enabling the real-time adaptation of pre-computed ROMs to new sets of parameters.

The main idea of the method is to represent each reduced-order basis built for a different parameter set as a point of the Grassmann manifold. One of these points is then taken as a reference point and all the other bases are mapped to the tangent space at this reference point (see Figure 1) . The interpolation process takes then place on this tangent space and the resulting interpolated quantity is mapped back to a new basis on the Grassmann manifold corresponding to a new set of parameters.

However, this method was limited by the fact that the chosen interpolation method was based on Lagrange polynomials that could only accommodate ROM variations with respect to one parameter. Here, it is shown that the nature of the presented ROM adaptation method allows the use of a much broader range of interpolation procedures.

As an illustration, this presentation exposes the extension of ROM adaptation to more realistic engineering situations where the ROM operating point depends on several physical parameters. For example, the CFD-based aeroelastic ROM of an aircraft can be fully modeled by two physical parameters,

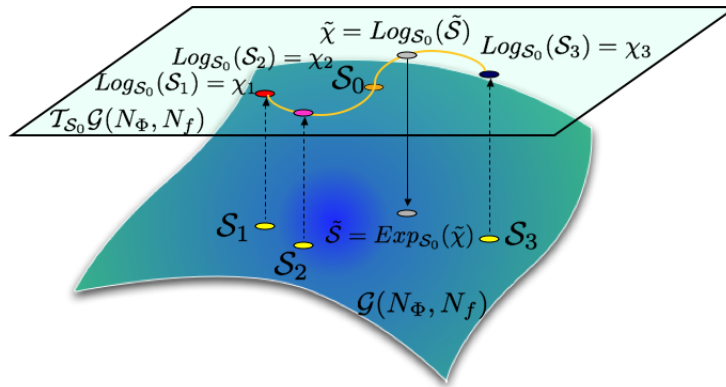


Figure 1: A graphical description of the interpolation of four subspaces in a tangent space to a Grassmann manifold.

namely the Mach number the aircraft is flying at, and its angle of attack. Thus, a multi-dimensional spline-based interpolation method is here presented for an extension of the original method to such multivariate adaptations. Full-aircraft aeroelastic ROMs of complete F-16 and F-18/A fighter configurations are here adapted to new flight conditions leading to very good correlations with results obtained from direct ROM constructions and full-order linearized simulations.

Another challenge that may be encountered in the process of interpolating reduced-order bases is the limitation on the number of previously computed ROMs accumulated in databases. While this may lead to low-order and thus poor accurate interpolated ROMs, very often, sensitivities of those pre-constructed bases can be computed at a very low cost at the same time these reduced-order bases are built. Thus, in order to tackle the previously mentioned issue, the authors present here as well a high-order ROM interpolation method that enables the use of information about the sensitivity of the reduced-order bases with respect to the modeling parameters. This method is applied to several physical mechanical systems, including a simplified mass-damper-spring system, showing that such an interpolation method can lead to very accurate results, compared to full-order responses, both in the time-domain and the frequency-domain.

As this method is applicable to ROMs based on projection schemes such as the popular POD method or Krylov subspaces-based ROMs, the potential of the proposed ROM adaptation method is very encouraging for near real-time aeroelastic predictions in the context of pre-computed ROM databases for realistic engineering applications.

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