Localized Collocation Meshless Method (LCMM) for Convectively Dominated Flows

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ABSTRACT

In this paper, we present the developments of a Localized Collocation Meshless Method (LCMM) to simulate fluid mechanics and heat transfer under laminar conditions. The meshless notion indicates that the current method is not cell-based; rather it relies on a point distribution in the computational domain. Each of those points is referred to as a data center. The preprocessing for this meshless method is independent from the geometry shape as it always yields a Cartesian point distribution in the interior and non-uniformly distributed points near the boundaries



(a) boundary data center. (b)an internal data center. Figure 1. The LCCM topology for data centers.

The formulation of LCMM depends on polynomial enriched radial basis function collocations and moving least square polynomial expansions. A localized expansion over a group or topology of influence points, NF, around each data center is sought for the dependent variable is ϕ

$$\phi(\boldsymbol{x}) = \sum_{j=1}^{NF} \alpha_j \chi_j(\boldsymbol{x}) + \sum_{j=1}^{NP} \alpha_{j+NF} P_j(\boldsymbol{x})$$
(1)

where α_j are unknown expansion coefficients, $\chi_j(\boldsymbol{x})$ are expansion functions, and NP polynomials $P_j(\boldsymbol{x})$ added to the expansion to guarantee that constant and linear fields can be retrieved exactly. The inverse Hardy Multiquadric $\chi_j(\boldsymbol{x})$ radial-basis [1] are used:

$$\chi_j(\boldsymbol{x}) = \left[r_j^2(\boldsymbol{x}) + c^2\right]^{-\frac{1}{2}}$$
 (2)

Where c is a shape parameter, and $r_j(\boldsymbol{x})$ is the Euclidean distance from (\boldsymbol{x}) to (\boldsymbol{x}_j) . We discuss the choice of c and he implementation of shadow points at the boundary, as well as the so-called "derivative vector" approach to express any order derivative at data center, whereby, a derivative is obtained by multiplying the corresponding derivative vector by a "scalar vector" comprises of the given field variable values at points neighboring the data center.

The Navier-Stokes equations with \vec{V} as the flow velocity vector,

$$\nabla \cdot \vec{V} = 0 \quad \text{and} \quad \rho \frac{\partial \vec{V}}{\partial t} + \rho \left(\vec{V} \cdot \nabla \right) \vec{V} = -\nabla p + \mu \nabla^2 \vec{V} \tag{1}$$

are explicit discretized using a third order time stepping scheme and the spatial derivatives are discretized utilizing a polynomial enriched radial basis function meshless method approach. A velocity correction iterative solution algorithm is utilized to ensure coupled satisfaction of all the equations at convergence [2]. As we dealing with convectively dominated fluid flow problems, a high order upwinding scheme is incorporated and a special scheme is developed to dampen the numerical oscillations. The meshless results were numerically validated using the finite volume commercial solver Fluent 6.2 and a research finite volume code. A very good agreement was found between the LCCM and the finite volume method as seen below for a natural convection problem in a rectangular cavity containing molten Aluminium: top and bottom adiabatic and temperature difference imposed between the two vertical walls inducing natural convection.



Figure 2. Meshless velocity vectors colored by magnitude for natural convection in a liquid aluminum cavity at 5s, 10, 20s, 25s, 30s and comparison of FVM and meshless Nusselt number evolution in time on the left-hand and right-hand walls of cavity.

References

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