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ESTIMATION OF SENSITIVITIES IN PARABOLIC BOUNDARY VALUE PROBLEMS BY STATISTICAL SIMULATION OF DIFFUSION PROCESSES [1]

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ABSTRACT

We consider boundary value problems of the forms

$$Lu + f(t,x,\vartheta) = 0, \quad t \in (0,T), \quad x \in G, \tag{1}$$
$$u(T,x,\vartheta) = \varphi(x,\vartheta), \quad x \in G, \tag{2}$$
$$u(t,x,\vartheta) = 0, \qquad x \in \partial G, \tag{3}$$

and

$$Lu + f(t,x,\vartheta) = 0, \qquad t \in (0,T), \quad x \in G, \tag{4}$$
$$u(T,x,\vartheta) = \varphi(x,\vartheta), \qquad x \in G, \tag{5}$$
$$\frac{\partial u}{\partial n} + \eta(t,x,\vartheta)u + \gamma(t,x,\vartheta) = 0, \quad x \in \partial G, \tag{6}$$

where $Lu \equiv \frac{\partial u}{\partial t} + \frac{1}{2}\sum_{i,j=1}^{d} b_{ij}(t,x,\vartheta)\frac{\partial u^2}{\partial x_i \partial x_j} + \sum_{i=1}^{d} a_i(t,x,\vartheta)\frac{\partial u}{\partial x_i} + c(t,x,\vartheta)u$; $B(t,x,\vartheta) = (b_{ij}(t,x,\vartheta))$ is a positively definite matrix; $G \subset R^d$ is a bounded connected domain with smooth boundary ∂G; n is a unit inward normal vector ; $\vartheta = (\vartheta_1,\ldots,\vartheta_m)$ is a vector of parameters.

It can be assigned a d-dimensional diffusion process $X.$ to the parabolic operator L, and the process $X.$ satisfies to a corresponding stochastic differential equation (SDE). The probabilistic representations of u at $(t,x) \in (0,T) \times G$ of the problems (1) — (3) and (4) — (6) are well known. These representations are appropriate functionals of stochastic process $X.$, that initiates at the point (t,x) and moves in G. To boundary conditions (3) and (6) correspond respectively absorbing and reflecting conditions of $X.$, when it reaches the boundary.

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Thus, to obtain estimation of $u(t,x)$ we need to simulate numerically a large number trajectories of the stochastic process $X.$ and calculate the approximate value of the appropriate functional.

In the work we construct the functionals of the processes $X.$ and $\frac{\partial X.}{\partial \vartheta_k}$ which are the probabilistic representations of $\frac{\partial u}{\partial \vartheta_k}$ $(k = 1, \ldots, m)$. To obtain the derivatives $\frac{\partial X.}{\partial \vartheta_k}$ we solve numerically SDE for $X.$ and SDE, which is derived from this by differentiation with respect to the parameters. The proof of convergence of obtained estimates of $\frac{\partial u}{\partial \vartheta_k}$ is given in the work. Some numerical experiments for model problems with known exact solution to estimation of the sensitivities $\frac{\partial u}{\partial \vartheta_k}$ and parameters ϑ are presented.