

## Finite element implementation of inter-elements discontinuities with zero elastic opening and plastic-damaging response.

\* Massimo Cuomo<sup>1</sup>

<sup>1</sup> University of Catania  
v.le A. Doria, 6  
95125 Catania - Italy  
mcuomo@dica.unict.it  
www.dica.unict.it

**Key Words:** *Interfaces, Strong Discontinuities, LCP, zero opening.*

### ABSTRACT

The FE analysis of the evolution and growth of discontinuities within a continuous media (either homogeneous or heterogeneous) can be performed in several ways: after the original methods based on some sort of spreading within the element have shown their objectivity problems, many forms of enrichment of the kinematic fields have been developed. Leaving apart the methods used in Fracture Mechanics, two major families can be recognized, that will be shortly denoted as embedded discontinuities and inter-elements discontinuities. While the former are largely more flexible, they still present limitation in the accuracy of the deformation field (SDA) or require special tracking procedure that cannot be set within the classical framework of FEM (XFEM).

Interelement discontinuities are rather rigid, since the possible paths of the interfaces must be known in advance, but they allow either a more accurate description of the real kinematics, and also the introduction of whatever cohesive interface that fit the experimental behaviour of the interface. Interelement interfaces are therefore also used as comparative test for other methods.

For the reasons outlined above many models of interface elements have been also recently introduced [1,2], often with the goal of enlarging the field of application and of modelling particular responses. However, unless special elements or constitutive behaviour are introduced, classical node-to node interface elements suffer of two major drawbacks: a fictitious elastic stiffness has to be introduced, in the absence of which node reactions, that determine the response of the element, cannot be directly obtained from the FE solution; the evaluation of damage in the response of the interface, that can be useful for accurately modelling unloading and friction, has to be compatible with the damage evolution in the continuum.

The paper takes the move from a recently contributed method for simulating crack opening using rigid-plastic interface elements, that avoids the introduction of interelement discontinuities until some opening criterium is met [3]. The corner reactions are accurately evaluated through a stress recovery algorithm [3] that has recently been extended to 3D. The structural problem that results is generally a Linear Complementarity one [4], and many algorithms can be used [see, e.g. 5,6]. These algorithms generally

require heavy matrix calculations, or ad hoc procedures that usually must be adjusted to the problem at hand.

In the present contribution it is proposed an alternative methodology for solving the structural problem of a continuum with interfaces that are active only when some opening criterion is met. The additional degrees of freedom are considered as Lagrange multipliers and an Augmented Lagrangian technique is employed, that exploits the fact that the number of additional degrees of freedom is usually much smaller than the number of the dof's of the continuum.

The development of the proposed methodology requires as starting point a mixed variational principle, accounting for elastic storage in the continuum and energy dissipation on the interface (and, eventually, in the continuum). Once the principle is formulated, its discretisation can be using classic displacement or mixed interpolations, similarly to what done in [7].

The paper will present the main development, and some comparison with alternative LCP formulations.

## REFERENCES

- [1] Alfano G, Crisfield MA. "Finite element interface models for the delamination analysis of laminated composites: mechanical and computational issues. *International Journal of Numerical Method in Engineering* **50**, 2001.
- [2] Ortiz M, Pandolfi A. "Finite-deformation irreversible cohesive elements for the three-dimensional crack propagation analysis". *International Journal for Numerical Methods in Engineering* **44** (9), 1267-1282, 1999.
- [3] Ciancio D, Carol I, Cuomo M. "On inter-element forces in the FEM-displacement formulation, and implications for stress recovery". *International Journal of Numerical Method in Engineering* **66**, 502-528, 2006.
- [4] Ciancio D, Carol I, Cuomo M. "Simulation of the opening of a crack through a corner node, using "rigidplastic" interface elements", Proceedings FRAMCOS 2007.
- [5] Tin-Loi F, Xia S.H. "Nonholonomic elasto-plastic analysis involving unilateral frictionless contact as a mixed complementarity condition", *Comp. Methods Applied Mech. Engineering*, **190**, 4551-4568, 2001.
- [6] G. Bolzon, G. Maier and G. Novati, "Some aspects of quasi-brittle fracture analysis as a linear complementarity problem". *Fracture and Damage in Quasi-brittle Structures* (Edited by Z. P. Bazant, Z. Bittnar, M. Jirasek and J. Mazars), pp. 159-174. Spon, London (1994).
- [7] Cuomo M, Ventura G. "Complementary Energy Approach to Contact Problems Based on Consistent Augmented Lagrangian Formulation", *Mathematical and Computer Modelling*, **28**, No.4-8, 185-204, 1998.