8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

High-order Discontinuous Galerkin Methods for the Reverse Time Migration

C. Baldassari [1,2], H. Barucq [1,2], H. Calandra [3], B. Denel [3], J. Diaz [1,2]

[1] INRIA Bordeaux Research Center
Team Project Magique3D

[1] LMA, CNRS UMR 5142
Université de Pau
64013 PAU. FRANCE

[1] TOTAL
Av Larribau
64000 PAU. FRANCE

Key Words: *Acoustic Wave Equation, Discontinuous Galerkin method, Seismic Imaging.*

ABSTRACT

In the framework of oil exploration, seismic processing is used to get a representation of the subsurface where the hydrocarbons are trapped. Since the work of Hemon [1] in the 70's, it is well-known from the geophysical community that the seismic imaging methods based on the solution of the full wave equation are the most efficient and accurate to deal with very complex propagation media, when compared to the ones based on the solution of the one-way wave equation. Nowadays one of the most commonly used imaging technique is the Reverse Time Migration (RTM), which relies on many successive solutions of the wave equation. Seismic imaging is thus requiring a very large amount of computational power, especially as far as the memory storage is concerned. Indeed accurate numerical methods for the wave equation lead to the solution of huge systems with several millions of degrees of freedom. It is therefore now crucial to develop low-storage consuming numerical methods while keeping the high accuracy required to obtain a good image of the subsurface. The wave equation is mostly solved by high-order finite difference methods in space which, when combined to a leap-frop-like scheme in time, leads to an explicit computation of the wave field since the mass matrix is diagonal. However finite differences are not well suitable to accurately take the topography effects into account because they involve grids which approximate the interface by steps. High order absorbing boundary conditions are not easy to implement in a finite difference scheme, especially since they require a specific treatment at the corners. Finite element methods are then an efficient alternative since they use meshes adapted to the topography of the domain and the boundary conditions are naturally taken into account in the variational formulation. Nevertheless, the most classical among them do not lead to an explicit scheme because the mass matrix (without any particular treatment) is not diagonal. To overcome this drawback, one can use the so-called mass lumping technique and compute the coefficients of the mass matrix by a judicious quadrature rule [2]. This technique generally penalizes the convergence order of the finite element method since the appropriate quadrature rules are mostly of lower order than the finite element method and that is why the spectral element method [3,4,5] has attracted geophysicists. The spectral elements are indeed compatible with the use of a Gauss-Lobato quadrature rule of the same order as the elements one. However, this method requires to mesh the domain with quadrangles in 2D ou hexaedra in 3D which are difficult to compute and not always suitable to complex topographies. Moreover, as far

as we know, the order of convergence of this method is not always guaranteed when the velocity field is varying inside each element. Recently, Grote et al. [6] applied the Interior Penalty Discontinuous Galerkin (IPDG) method to the wave equation. This new approach is based on meshes made of triangles in 2D or tetrahedra in 3D, which implies the topography of the computational domain is discretized accurately. Furthermore, the resulting discrete scheme remains quasi-explicit because the mass matrix is block-diagonal without any approximation of its entries by a quadrature rule. The convergence order is then ensured as for the spectral element method. Moreover this method seems to be well-adapted for the handling of polynomial velocities inside each element. In this work, we will present the IPDG method we use and we will compare it to a spectral element method in the1D case. Then we will validate it and analyze its performance for the RTM in 2D. We will also consider various time discretization schemes in order to evaluate the influence of the time integration on the computation of the solution in complex media. We will particularly take care of the dispersion effects produced by the CFL condition which has to be adapted to the velocity discontinuities. The reference solution are computed either analytically by the Cagniard-de Hoop method for simple configurations or numerically by a finite difference scheme of eighth order in space and second order in time for more complex problems.

REFERENCES

[1] C. Hemon. “Equations des ondes et modèles”. *Geophysics Prospecting*, Vol. **26**, 790–821, 1978.

[2] P.G. Ciarlet. “Finite Element Method for Elliptic Problems”. North-Holland, Amsterdam, 1978.

[3] G. Cohen, P. Joly and N. Tordjman. “Higher-order finite elements with mass-lumping for the 1D wave equation”. *Finite Elements in analysis and Design*, Vol. **16**, Issues 3-4, 329–336, 1994.

[4] G. Seriani and E. Priolo. “Spectral element method for acoustic wave simulation in heterogeneous media”. *Finite Elements in analysis and Design*, Vol. **16**, Issues 3-4, 337–348, 1994.

[5] D. Komatitsch and J. Tromp. “Introduction to the spectral element method for three dimensional seismic wave propagation”. *Geophys J. Int.*, Vol. **139**, 806–822, 1999.

[6] M. J. Grote, A. Schneebeli and D. Schötzau. “Discontinuous Galerkin finite element for the wave equation”. *SIAM J. Numer. Anal.*, Vol. **44**, no 6, 2408–2431.