8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

EXTENDING THE PHILOSOPHY OF SCIENCE TO THE PHILOSOPHY OF COMPUTATIONAL ENGINEERING. THE FALSIFICATION ROLE OF CONVERGENCE ANALYSIS

***E. R. de Arantes e Oliveira[1]**

[3] Department of Civil Engineering, Instituto Superior Técnico
Av. Rovisco Pais, Lisbon, 1049-001, Portugal
arantes@civil.ist.utl.pt

Key Words: *Philosophy of Science, Computational Experimentation, Falsification, Convergence*

ABSTRACT

Few theories have progressed so much in the second half of the XXth century as the theory of structures.

The advent of computers was certainly the main cause of a fast evolution, which started in the mid-forties with the matrix methods, and grew faster in the mid-fifties after the finite element techniques emerged in the field. The change was however deeper than a mere adaptation to computers. In the field of structures, as in other fields, such adaptation was indeed the starting point for a complete reformulation that removed most of the imperfections that now appear so obvious. The modern theory of structures (Arantes e Oliveira, 1970, 1973, 1975) is no longer a collection of methods of analysis. It is even more than a collection of different analogous, continuous or discrete, mathematical models which emulate the mechanical behaviour of the different kinds of structures. It became indeed a hierarchic set of models successively generated with the help of systems of rules "justified" by convergence theorems. In other words, the generated models are falsifiable (i. e., empirically refutable, in which the word empirically has to do with computational experimentation) by the same kind of convergence analyses that had so far been used for series solutions.

The basis of the theory is the three-dimensional model provided by the mechanics of solids, which can be applied to any kind of solid body. The three-dimensional model is thus privileged, not because it is more consistent in itself, but because it is believed to provide a better simulation of the mechanical behaviour of solid bodies. For this reason, and not for any other, it is usually regarded as the fundamental model which generates all the others. Then, a general model is introduced. Within its frame, the energy and variational theorems common to all structural models are stated and demonstrated, and the generating rules are given that permit the generated models to be obtained from the generating ones. The two and three-dimensional models may then be generated from the three-dimensional model, which are the basis of the theories of shells and rods, and the discrete models follow which incorporate some classical results and methods of the old theory of structures, as well as the basis of the theory of finite elements and of other discretisation techniques. As stated above, convergence is the justification for the

generating methods and provides the possibility of falsifying any model of the theory of structures.

The need for a falsification criterion was felt more strongly when the energy methods started to be used for generating discrete models with the help of the finite element technique, as it became clear that such discrete models were to be rejected if the sequences of approximate discrete solutions obtained by considering successive subdivisions of the body into finite elements with indefinitely decreasing dimensions did not converge to the exact solutions. The role of convergence is thus well known in case of discrete models, like those generated by the finite element technique, but not so well known in the generation of continuous models, like the theory of shells and rods. For these, however, the convergence test must be applied in a different way. In order to clarify this point, let us consider the generation of the theory of shells.

Although the theory of shells is usually presented as resulting from the two-dimensional model generated from three-dimensional elasticity, it is well known that direct approaches are possible, i. e., that the shell equations can be established without referring to the three-dimensional theory. Most specialists in the field have however disliked direct approaches for the derivation of shell equations. Indeed, although they can provide the right equations, they give no information about the connection between the solutions provided by those equations and by the three-dimensional ones. Mathematicians have preferred, therefore, to derive the shell equations from the three-dimensional model, by introducing approximate assumptions. A great deal of research has been inspired by the wish to improve and control such approximation procedure. Many researchers also put energy methods aside because, in the past, they were presented practically as direct methods, in the sense that the connection between the solutions they lead to and the corresponding three-dimensional solutions had not been investigated. In other words, the virtual work principle had been used, as well as variational theorems, but such use had not been justified.

In our synthetic vision of the theory of structures, the theory of shells appears as a two-dimensional model generated from the three-dimensional one. The convergence concepts and theorems must be used in order that it can be declared as a valid approximation. As told above, the situation is however not quite the same as in the case of the finite element method. As a matter of fact, in the case of the latter, a sequence of approximate solutions must converge to the exact one, while in the theory of shells what must be investigated is the convergence of a sequence of three-dimensional solutions to a two-dimensional solution, i. e., the convergence of a sequence of exact solutions to the approximate one. Indeed, the crucial point is proving that the two-dimensional solution approaches more and more the corresponding three-dimensional ones as the shell becomes thinner and thinner, provided the relative values of the bending and membrane stiffness coefficients do not change as the thickness tends to zero. Such condition cannot be respected in a classical shell in which the bending moments result merely from the ordinary stresses distributed in the thickness t , as the bending stiffness coefficients are then proportional to t3, and the membrane stiffness ones simply to t . But it can be satisfied in a generalised Cosserat's shell in which the couple-stresses are not supposed to vanish.