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Stationary Analysis by a Thermal Convection Solver with Hierarchical Domain Decomposition Method

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ABSTRACT

There are often encounter requirements to compute which flow pattern is generated in stationary state. With progress of computer environment and increasing demand of precise analysis, numbers of degrees of freedom for such computations become larger. However, as far as we know, computational codes, which are efficient for large scale, stationary and nonlinear flow problems, are rare. Therefore, we have developed ADVENTURE_sFlow [3], which is one of the modules produced in the ADVENTURE project [1].

ADVENTURE_sFlow uses the Newton method as a nonlinear iteration, and to compute the problem at each step of the nonlinear iteration a stabilized finite element method is introduced. Moreover, to reduce the computational costs, an iterative domain decomposition method is applied to stabilized finite element approximations of stationary Navier–Stokes equations, where Generalized Product-type method based on Bi-CG (GPBiCG) [6] is used as the iterative solver of the reduced linear system in each step of the nonlinear iteration. A parallel computing using the Hierarchical Domain Decomposition Method (HDDM)[2][4][5] is also introduced.

This paper describes a finite element method with stabilization technique used to solve Navier-Stokes equations, and the advection-diffusion equation for thermal convection problems with the Boussinesq approximation. We use P1 elements for the velocity, the pressure and the temperature.

Equations:

$$(\mathbf{u}\cdot\nabla)\mathbf{u} - 2\nu\nabla\cdot D(\mathbf{u}) + \nabla p = \mathbf{g}\{1-\beta(T-T_r)\} \qquad \textbf{in}\,\Omega$$

$$\nabla\cdot\mathbf{u} = 0 \qquad \textbf{in}\,\Omega$$

$$\mathbf{u}\cdot\nabla T - a\Delta T = S \qquad \textbf{in}\,\Omega$$

where $\mathbf{u} = (u_1, u_2, u_3)^T$: velocity vector[m/s], ν : kinematic viscosity coefficient[m^2/s], p : normalized gauge pressure[m^2/s^2], $\mathbf{g}$: gravity[m/s^2], β : coefficient of thermal expansion[$1/K$], T : temperature[K], T_r : reference temperature[K], a : thermal diffusion coefficient[m^2/s], S : source term[K/s], D_{ij} : the rate of strain tensor [1/s] and

$$D_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

Results: Figure 1 shows the theoretical analysis about a well known thermal problem. We compare these computed data with the theoretical ones. Figure 2 shows results of a thermal cavity problem.

Conclusions: To analyze the stationary Navier–Stokes equations, ADVENTURE_sFlow was developed, and is one of the modules produced in the ADVENTURE project [1]. This time, this solver has been applied to thermal convection problems with good performance.

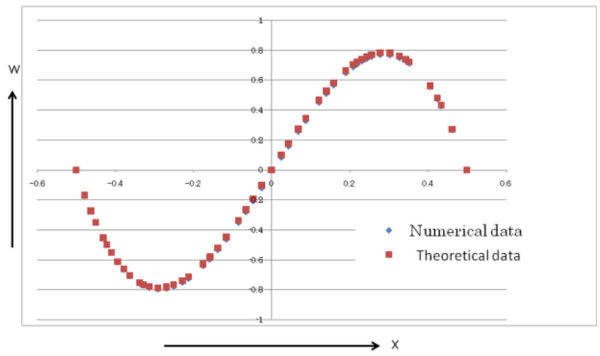

Fig.1 Comperision between numerical data and theoretical ones

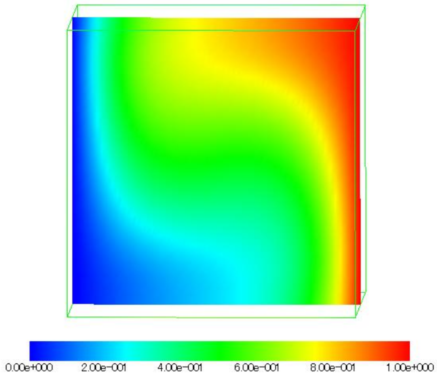

Fig.2 Results of a thermal cavity problem

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