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MULTISCALE BUCKLING ANALYSIS OF HETEROGENEOUS MATERIALS

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ABSTRACT

In this work, we propose an efficient numerical technique to analyze the post-buckling of heterogeneous nonlinear materials. The present technique is able to handle instabilities at both macro and micro levels. Modeling multiscale buckling of heterogeneous materials is a challenging task. In the nonlinear context, the effective properties of the media can not be obtained through the techniques used for linear media, as the superposition principle no more holds. Thus, one alternative to meshing the whole structure, including all heterogeneities is to use a multilevel finite element (FE^2) [1,2] procedure to extract from a microscopic problem associated with a representative volume element (RVE) the macroscopic stresses at every material (Gauss point) of the macroscopic structure. The nonlinear problems at the micro and macro levels are usually solved through classical Newton-Raphson procedures, which are known to break down in presence of instabilities.
To address the aforementioned issue, we combine the multilevel finite element analysis and the Asymptotic Numerical Method (ANM) [3-4]. The ANM is based on an asymptotic expansion of the solution field. Introducing such expansion in the nonlinear problems at both macro and micro scales, it results a series of linear problems that must be solved successively. Compared to the FE^2 method, the proposed technique offers the following advantages: (i) in the context of a multilevel finite element analysis, the solutions of the nonlinear microscopic problems are obtained by solving series of linear problems. One straightforward consequence is that a localization tensor can then be constructed for each order of the asymptotic development and an explicit constitutive law can then be constructed at the macroscopic level at each order. Thus, while in the FE^2 method the linearization is only valid at each Newton-Raphson step, in our procedure the linearization remains valid along the whole asymptotic step; (ii) in the ANM, the response curve is explicitly represented in the form of a vectorial series which is calculated with only one decomposition of the stiffness matrix. This representation with series contains several informations that are inaccessible by the classic algorithms. A great advantage easily is not drawn: the length of each step of calculation is a posteriori estimated; (iii) The ANM is a very simple technique for solving problems involving junctions and instabilities, as compared to Newton-Raphson-based methods, where an arc-control procedure is required. In the context of multiscale analysis, such technique would be computationally inefficient.
We have examined the accuracy and efficiency of the proposed technique through

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several problems involving buckling of hyperelastic heterogeneous structures. We have compared the solution of our multiscale procedure with the solution of a fully meshed structure, involving the explicit mesh of all heterogeneities. Different RVE with different shapes, volume fractions and materials properties have been used. Satisfactory results have been obtained with a drastic reduction of the computational costs.
To evaluate the capability of the Multiscale-ANM method to handle severe buckling and instabilities at both macro and micro levels, we have studied several problems. Firstly, the buckling of a shallow arch made of a heterogeneous material submitted to a concentrated load at its middle, using three types RVE (see figure 1a) is investigated. The displacements of the applied force point versus the loading parameter for three types of RVE are presented in figure 1b. The volume fractions are the same in all RVEs. It is worth noting that the response curve is obtained with only two asymptotic steps. Secondly, we have studied the compression of a fiber reinforced composite material, where instabilities occur at the microscale.

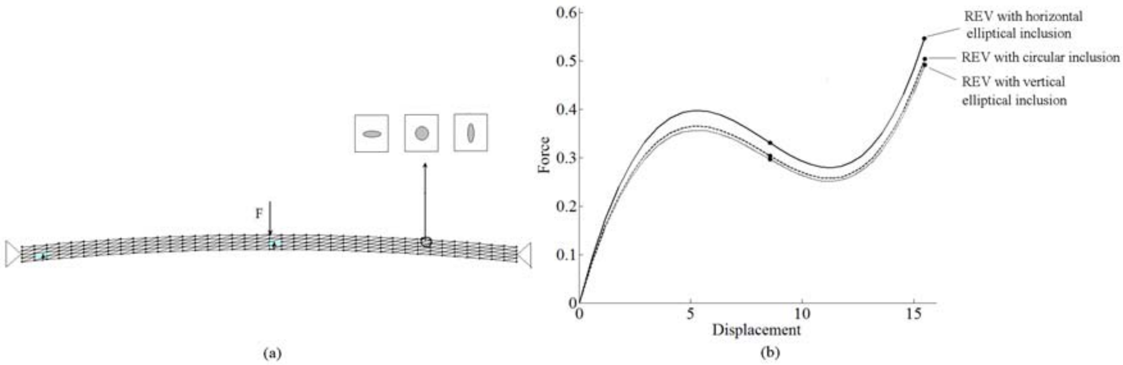

Figure 1. (a) Geometry of the arch problem; (b) Force- Displacement response for arch;

As a conclusion, the proposed technique provides an efficient framework to analyze the post-buckling regime of heterogeneous materials, where the instabilities may occur at both the macro and micro levels.

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