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CONSTITUTIVE MODELING FOR GEOTECHNICAL MATERIALS USING THERMODYNAMIC POTENTIALS: CLASSICAL DRUCKER-PRAGER AND DRUCKER-PRAGER CAP MODEL

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ABSTRACT

The standard procedure to construct elastic-plastic models of soils, clays and other geomaterials is to specify the elasticity law, yield condition, flow rule and hardening law all independently of each other [3]. Recently, another approach based in the theory of thermomechanics with internal variables is being used in the constitutive modeling of geomaterials. In this approach, the whole of the constitutive structure (yield condition, flow rule, hardening laws and elasticity law) is determined from just two thermodynamic potentials [3]: an energy potential and a dissipation function. The purpose of this work is to present a formulation for the constitutive modeling of geotechnical materials based on the use of two thermodynamic potentials with the help of convex analysis, applying the theory presented to the classical Drucker-Prager Model and to the Drucker-Prager Cap Model. The constitutive relationships are obtained by means of the energy potential, while the flow laws are derived from the dissipation function. Internal variables are used to handle plasticity with hardening. In this case, the flow rule is replaced by an evolution law that relates the rate of change of internal variables and the conjugate thermodynamic forces [5]. Convex analysis techniques are applied [7], making it possible to derive two equivalent forms of the plastic flow law. The initial-boundary value problem is formulated in two alternative ways depending on which of the forms of the flow law is adopted [8]. The thermodynamic potentials for the classical Drucker-Prager Model (without hardening) and for the Drucker-Prager Cap Model are presented. The constitutive problem is written as a non-linear complementarity problem and solved with a non-linear mathematical programming algorithm. A finite dimensional optimization problem is identified in the minimum principle derived from the time-discretized version of the variational formulation of the cinematic initial boundary value problem [4]. Therefore, a solution procedure, combining mathematical programming algorithms to solve the constitutive problem and

the Quasi-Newton Method to the equilibrium problem, is adopted. Spatial discretization is performed by means of the Finite Element Method [6] and time discretization is implemented independently. Two examples are given to illustrate the proposed approach and the results of the computations are compared with experimental and numerical results from literature. The good agreement of the numerical results obtained in this work and the numerical and experimental results presented in literature contributes to validate the numerical models implemented.

The constitutive behavior of an elastic-plastic medium can be described in terms of scalar and tensor fields which properties must conform to the laws of continuum thermodynamics [1]. One of the advantages of the present approach is that the results obtained satisfy the fundamental laws of thermodynamics. In addition, once the constitutive model is cast in a convex analytic setting, the restriction to smooth yield surfaces is dropped and plastic deformations can be determined in singular points of the yield surface without the use of special techniques. In this aspect, the formulation proves to be advantageous as long as it eliminates the expense of determining the values of additional parameters used in literature in order to obtain smooth yield surfaces. Finally, in the present work a parabolic cap is used in the Drucker-Prager Cap Model [2] instead of an elliptic cap. For the example in analysis, it was observed that the substitution of the elliptic cap by a parabolic cap hasn't modified the evolution of the body response in terms of displacements in the set of prescribed load steps.

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