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VOCAL TRACT SHAPE DESIGN PROBLEM BASED ON RESONANT POLES FOR THE HELMHOLTZ EQUATION

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ABSTRACT

The voice generation process of human vowels is model mathematically by the Helmholtz equation in an open unbounded region with source term located on the boundary that corresponds to the vocal cords position. Introducing the Dirichlet-to-Neumann mapping on the artificial boundary that separates the interior vocal tract region and the exterior region, we can compute the frequency response function with corresponding formant peaks.

To simulate numerically the individual human vowels, it is essential to compute the frequency response function that is defined to be the mapping F from the wave number k or the time frequency appearing in the Helmholtz equation to the pressure intensity at some exterior observation point x:

$$F(\omega, x) = p(x, \omega),\ \ \omega = ck$$

with the solution p of the Helmholtz equation:

$$(1)\quad -c^2 \Delta p(x,\omega) = \omega^2 p(x,\omega).$$

We impose the outgoing radiation at infinity in the exterior region to single out the physical solution and assume the rigid wall condition on the interior boundary of vocal tract except on the sound source position where we impose the incident time harmonic inhomogeneous Neumann boundary condition.

The peaks of the frequency response function are related to the complex resonant eigenvalues corresponding to the non-trivial solution for the Helmholtz equation (1) with homogeneous boundary condition.

By designing the positions of complex resonant eigenvalues, we can control the formants, namely the peaks of the frequency response function that characterize the individual vowels. In the previous works [1], [2], [3] and [4], we studied the designing problem using mainly the one dimensional model called Webster's horn equation.

In this talk, we extends the method to the two dimensional case and use the variational formula of the complex eigenvalues for optimising the positions of the resonant poles to

match the positions of the formants to those of the target frequency response function given by the target vocal tract shape. To implement this algorithm practically, we restrict the deformation of the vocal tract shape to be a linear combination of a finite number of basis smooth functions.

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