8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A SEGMENT-TO-SEGMENT CONTACT SMOOTHING METHOD FOR FINITE DEFORMATION

*** Manuel Tur[1], Javier Fuenmayor[2] and Peter Wriggers[3]**

[1] U.P. Valencia
Camino de Vera s/n
46022 Valencia (Spain)
matuva@mcm.upv.es

[2] U.P. Valencia
Camino de Vera s/n
46022 Valencia (Spain)
ffuenmay@mcm.upv.es

[3] IBNM-Universitat Hannover
Appelstraße 9a
30167 Hannover (Germany)
wriggers@ibnm.uni-hannover.de

Key Words: *Contact smoothing, Segment-to-segment, Mortar method.*

ABSTRACT

The so called mortar method has been successfully applied to solve frictionless and frictional contact problems using different methods as the penalty, Lagrange multiplier or augmented Lagrange method [1-2]. Unlike the classical node-to-segment method, the mortar or segment-to-segment method allows for an optimal convergence rate for nonconforming meshes.

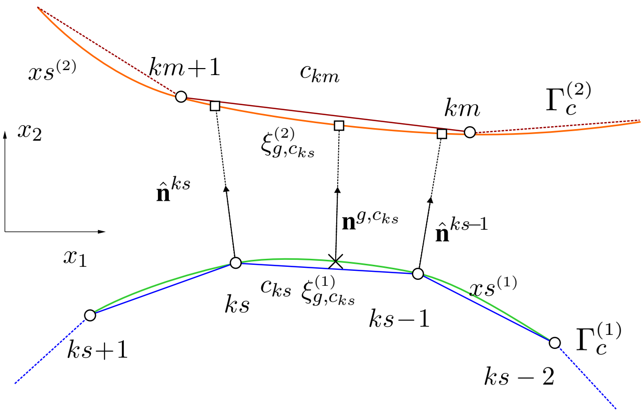


Figure 1: Smoothed surfaces using Hermite polynomials.

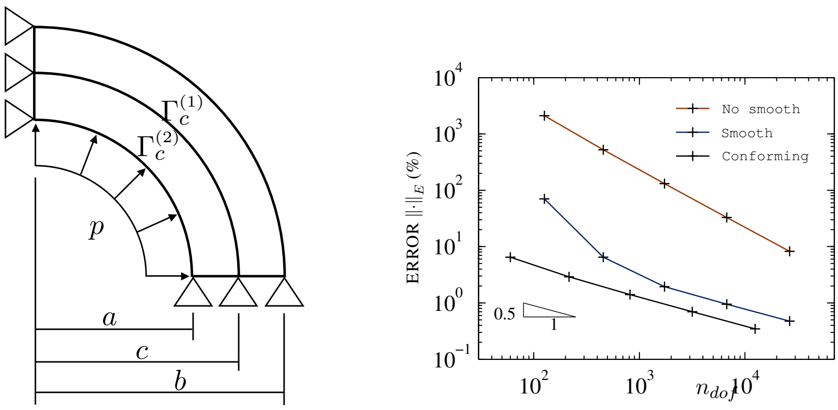
In this work we propose a segment-to-segment formulation where the contact surfaces of the bodies (master and slave) are defined using cubic Hermite polynomials. The contact problem is formulated with the Lagrange multiplier method and the following variational equation is obtained [1]:

$$
\begin{aligned}
&\delta W^i(\mathbf{v}^h, \mathbf{u}^h) + \int_{\Gamma_c^{(1)}} \lambda_N^h \, \delta g_N^h \, d\Gamma = \delta W^e(\mathbf{v}^h) \\
&\int_{\Gamma_c^{(1)}} \delta\lambda_N^h \, g_N^h \, d\Gamma \geq 0
\end{aligned}
\tag{1}
$$

where g_N is the gap, δg_N is the virtual gap, λ_N is the Lagrange multiplier (contact pressure) and $\delta\lambda_N$ is the variation of the multiplier. $\delta W^i(\mathbf{v}^h, \mathbf{u}^h)$ is the virtual work due to internal forces and $\delta W^e(\mathbf{v}^h)$ is the virtual work due to external forces. Linear elements are considered and a linear interpolation is defined for the multiplier and its variations.

For a given segment (see figure 1), the smooth definition of the surface (C^1 cubic polynomial) is done using the position of the two nodes defining the segment $\mathbf{x}^{ks-1}$ and $\mathbf{x}^{ks}$ and the averaged normal vectors $\hat{\mathbf{n}}^{ks-1}$ and $\hat{\mathbf{n}}^{ks}$. Since numerical integration is performed in order to compute the contact integrals, each contact unknown (gap, multiplier and its variations) is calculated for a given integration point taking into account the cubic Hermite polynomial.

A number of numerical examples have been solved for small and large deformation. For example, the problem of a cylinder under internal pressure, depicted in figure 2(a), has an analytical solution so the discretization error can be computed. The problem has been solved with a sequence of uniformly refined meshes. In figure 2(b) the error in energy norm is plotted as a function of the number of degrees of freedom of the mesh. Three cases are compared: a conforming mesh, a formulation without smoothing and the proposed smooth contact. Note that, although the initial gap is known to be zero in this problem, it has been calculated from the position of the nodes (like in a large deformation problem).



(a) Schematic model (b) Error in energy norm

Figure 2: Cylinder under internal pressure.

REFERENCES

[1] K.A. Fischer and P. Wriggers, "Frictionless 2D contact formulations for finite deformations based on the mortar method", *Comput. Mechs.*, 36, 226–244, 2005

[2] B. Yang, T.A. Laursen, X. Meng, "Two dimensional mortar methods for large deformation frictional sliding", *Int. J. Numer. Meth. Engrg.*, 62, 1183–1225, 2005