

## Some remarks on quadrilateral finite elements

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### ABSTRACT

It is well known that quadrilateral finite elements may produce unsatisfactory results when used on distorted meshes. Interested audience is referred to [1] and [2] for a rigorous mathematical analysis of this phenomenon. It turns out that many commonly used finite elements achieve suboptimal convergence properties on distorted quadrilaterals; among such elements we recall in particular serendipity (trunk) scalar elements and basically all vectorial elements for the approximation of problems involving the functional space  $H(\text{div})$  (like Raviart–Thomas or Brezzi–Douglas–Marini spaces for Darcy flow).

On the other hand, mimetic finite differences have become popular for the approximation of problems involving  $H(\text{div})$  on very general geometries. We refer to [3] and to the references therein for the description and the convergence analysis of mimetic finite differences.

The aim of this communication is to show how to use the ideas of mimetic finite differences for the stabilization of Raviart–Thomas element on general quadrilateral meshes. It turns out that such stabilization can be performed by a slight modification of the standard Raviart–Thomas element which does not increase significantly the computational cost of the original element.

### REFERENCES

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