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CALIBRATION OF THIRD ORDER ELASTIC MODULI BY THE ECHO PULSE METHOD

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ABSTRACT

Acoustoelasticity describes stress dependency of the velocities of sound-like waves propagating through an elastic material. This phenomenon may be used of in the echo pulse method to determine material parameters (moduli) and both the residual and active stresses in elastic materials and structures.

The theory of acoustoelasticity was introduced by Hughes and Kelly [1]. They measured the effect of the uniaxial stress on the velocity of the sound wave in an isotropic elastic material. They also proposed the method for identification of the three additional third-order elastic constants corresponding to Murnaghan's free energy function [2]. Later on, Thurston and Brugger [3] extended the theory to the case of an anisotropic material with arbitrary symmetry, which was further developed by several authors. However, all of them based their equations on the Green-Lagrange strain tensor.

The frequently used Green-Lagrange strain tensor is easy and straightforward in its definition (it is the pull-back of the covariant current metric tensor) and may well be used, the constitutive models based on it often exhibit definite instabilities when performing under large deformation (see [4]). For example, the two-parameter elasticity rotations with Green-Lagrange strain tensor give completely non-realistic material response already beyond 30 percent strain. In acoustoelasticity, this undesirable feature leads to high sensitivity of the third order material parameters to small perturbations applied to the input velocities (which may be thought of as measurement uncertainty). For experimental evidence performed on annealed and non-annealed aluminium alloys, see [5].

Theoretically, all the strain measures are equivalent but for a fixed choice of the stored energy function, for instance a polynomial of the third-order, different strain tensors will represent different material models [6]. If so, the definition of the strain tensor becomes essential and, in particular, stability then seems to be the key issue. Because of these reasons and keeping stability issues in mind, the authors of this paper set out to derive acoustic tensors based on the third order polynomial free energy function, using an arbitrary Seth-Hill strain measure, seeking the best alternative in terms of sensitivity property.

It turned out that among all possibilities the Hencky logarithmic strain tensor performed best. In this work, the coefficients of the acoustic tensor as functions of the applied initial stress are determined.

The result may be used to establish numerical values of material constants suitable for the logarithmic model of hyperelasticity.

Resulting expressions for the elastic moduli identifications and wave velocities in a homogeneous deformation field induced by three simple modes of pre-stress

- pre-stress by hydrostatic pressure p

$$\rho_0 c_{\mathrm{L}}^2 = \Lambda + 2\mu - \frac{p}{3K}\left(7\Lambda + 10\mu + 6l + 4m\right)$$
$$\rho_0 c_{\mathrm{S}}^2 = \mu - \frac{p}{3K}\left(3\Lambda + 6\mu + 3m - \frac{1}{2}n\right)$$

- material pre-stressed in longitudinal direction $\sigma_{xx} = t$

$$\rho_0 c_{\mathrm{L}}^2 = \Lambda + 2\mu + \left[\Lambda + 2l + \left(4\Lambda + 10\mu + 4m\right)\frac{\Lambda + \mu}{\mu}\right]\frac{t}{3\Lambda + 2\mu}$$
$$\rho_0 c_{\mathrm{S}}^2 = \mu + \left[4\Lambda + 4\mu + m + \frac{n\Lambda}{4\mu}\right]\frac{t}{3\Lambda + 2\mu}$$

- material pre-stressed in transverse direction $\sigma_{yy} = t$

$$\rho_0 c_{\mathrm{L}}^2 = \Lambda + 2\mu + \left[2l - \frac{2\Lambda}{\mu}\left(\Lambda + 2\mu + m\right)\right]\frac{t}{3\Lambda + 2\mu}$$
$$\rho_0 c_{\mathrm{S1}}^2 = \mu + \left[m - 2\Lambda - \frac{n\left(\Lambda + \mu\right)}{2\mu}\right]\frac{t}{3\Lambda + 2\mu}$$
$$\rho_0 c_{\mathrm{S2}}^2 = \mu + \left[\Lambda + 2\mu + m + \frac{n\Lambda}{4\mu}\right]\frac{t}{3\Lambda + 2\mu}$$

It is shown that the coefficients going with Hencky's strain definition are thrice less sensitive than those of the original Hughes and Kelly solution.

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