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The finite element immersed boundary method: model, stability, and applications

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ABSTRACT

The finite element approach to the immersed boundary method has been recently introduced and investigated [1-5] proving to have some potential advantages with respect to the original finite differences based one. In particular, no approximation of the Dirac delta functions is needed, thus allowing for a less diffusive treatment of the boundary interface. This fact has the effect of intrinsically minimizing the volume loss thanks also to the possibility of using very effective mixed finite elements for the approximation of the fluid equations; in general, discontinuous pressure schemes allow for a very good local enforcement of the divergence free constraint and for an optimal discretization of pressure jumps.

Numerical results in two and three dimensions fully confirm the good performance of our scheme; a preliminary stability analysis shows a condition linking the time step size and the discretization parameter along the immersed boundary. This condition is confirmed by our numerical experiments.

The immersed boundary method can be used for several applications ranging over different fields. A challenging goal is the simulation of the cochlea. The tube of the cochlea is divided by membranes, the most important of which is the ”basilar membrane” whose vibration transmits the sound waves along the tube. From a mechanical point of view, the basilar membrane behaves like a thin plate so that its displacement can be described by means of the Kirchhoff-Love model. The immersed boundary method takes into account the presence of the membrane immersed in the fluid, by means of an additional force density term, which is added to the right hand side of the momentum equation. Hence a finite element representation of the force generated by the Kirchhoff-Love model is needed. While there is a wide literature on the problem of solving the Kirchhoff-Love model given the force, the ”inverse” problem of computing the force given the displacement is less studied and turns out to be a nontrivial task. Our experiments show that the method can be unstable, unless a mass lumping is introduced.

The aim of this communication is to review the main ingredients of the finite element immersed boundary method and to address the major issue for a full simulation of the cochlea.

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