8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Airfoil performance at stochastic transonic flow regimes

J.-C. Chassaing[1], D. Lucor[1]

[1] Université Pierre et Marie Curie, Institut Jean Le Rond d'Alembert
Case 162, 4 Place Jussieu, 75252 Paris Cedex 05, France
jean-camille.chassaing@upmc.fr, lucor@lmm.jussieu.fr

Key Words: *uncertainty quantification, probabilistic CFD, Polynomial Chaos, transonic flow*

ABSTRACT

Probabilistic CFD and FSI methods are increasingly used for uncertainty quantification (UQ) of incompressible and compressible flow simulations due to inherent random parameters [1-2]. For transonic or supersonic compressible flows, the mean-flow structure changes can be explained by the analysis of the shock wave stochastic response [3]. In this study, a deterministic compressible RANS solver is coupled to a non-intrusive stochastic collocation solver to propagate several aerodynamic uncertainties through a transonic steady flow around a NACA0012 airfoil. The uncertain parameters under consideration in the numerical simulations are the free stream Mach number (M_∞), the angle of attack (AoA) and the turbulence intensity at the far field. The variability of these parameters can be attributed to inherent randomness due to freestream turbulence and/or aeroelastic deformations of the structure.

The objective of this work is twofold. First, we aim to demonstrate the efficiency and robustness of the non-intrusive stochastic collocation approach for uncertainty quantifications with flows presenting shock-waves and separation regions. Second, this study intends to highlight the potential importance of considering multiple uncertainties, with possibly different random distributions, on the stochastic response of the system. Toward this end, a complete physical analysis of the statistical system response is performed in the case of uncertain M_∞ and AoA with *uniform* random distributions. The distributions are characterized by $\mathrm{M}_\infty = 0.7 \pm 9\%$; $\mathrm{AoA} = 6\ \mathrm{deg} \pm 4\%$. The stochastic collocation method, based on a generalized Polynomial Chaos representation, consists in projecting directly the stochastic solution onto each member of the orthogonal basis chosen to span the random space. The non-intrusive approach does not require any substantial modifications to the deterministic solver. The evaluation of the solution moments is equivalent to computing multi-dimensional integrals over the probability domain. Different ways of dealing with high-dimensional integrations can be considered depending on the prevalence of accuracy versus efficiency. Here, we use a numerical quadrature of Gauss-type by full tensor products. This approach is very accurate and remains computationally efficient for a moderate number of random dimensions. In this study, we use twelve quadrature points per random dimension. The deterministic steady-state flow solutions are computed at a Reynolds number $Re = 5.5 \times 10^6$ using a Favre-Reynolds-averaged Navier-Stokes solver with a $O(\Delta x^3)$ upwind-biased scheme and Reynolds-stress model [4].

Then, the stochastic response surfaces of the aerodynamic lift and drag coefficients and their associated probability density functions can be computed. The statistical pressure distribution around the airfoil (fig. 1) and the coefficient of variation of the Mach number (fig. 2) show large variability which reveals the sensitivity of the shock wave and flow separated regions. Furthermore, it has been found from a sensitivity analysis (Sobol's coefficients) that there exists a significant coupling between the uncertainties

acting on the lift coefficient near off-design conditions. In addition to a full presentation of the convergence properties of the method applied to the studied configuration, the final abstract will include the effects of different random distributions (Gaussian, uniform, gamma) per direction of uncertainty and a parametric stochastic analysis of the freestream turbulence intensity whose potential influence on the shock-wave position and strength will be highlighted.

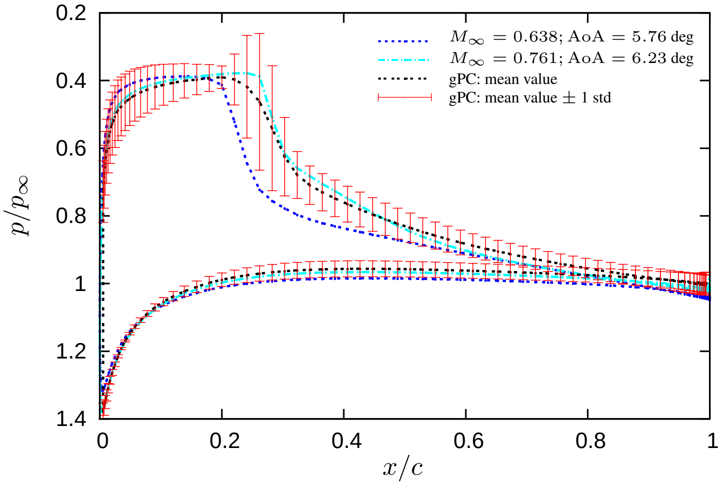

Figure 1: Non–dimensional steady–state pressure distributions around the NACA0012 airfoil. Comparison between deterministic computations ($\mathrm{M}_\infty = 0.638$; $\mathrm{AoA} = 5.76$ deg. and $\mathrm{M}_\infty = 0.761$; $\mathrm{AoA} = 6.23$ deg.) and statistical moments (mean solution and error bars) where the random parameters have *uniform* distributions with $\mathrm{M}_\infty = 0.7 \pm 9\%$; $\mathrm{AoA} = 6$ deg $\pm 4\%$

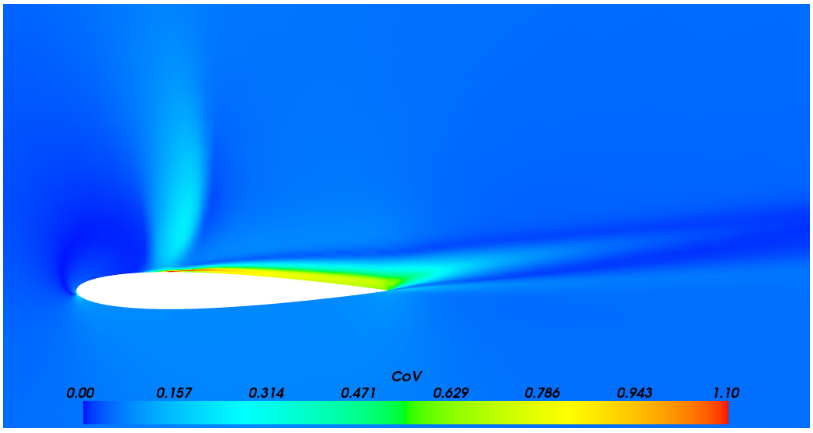

Figure 2: Mach number coefficient of variation σ/μ for uncertain angle-of-attack (AoA) and freestream Mach number (M_∞) with *uniform* random distributions ($\mathrm{M}_\infty = 0.7 \pm 9\%$; $\mathrm{AoA} = 6$ deg $\pm 4\%$)

REFERENCES

[1] D. Lucor, D. Xiu, C.-H. Su and G. E. Karniadakis. "Predictability and uncertainty in CFD", *Int. J. Numer. Meth. Fluids*, Vol. **43**, 483–505, 2003.

[2] G. J. A. Loeven, J. A. S. Witteven and H. Bijl. "Probabilistic collocation: an efficient non-Intrusive approach for arbitrarily distributed parametric uncertainties", AIAA Paper 2007-317.

[3] G. Lin, C. -H Su and G. E. Karniadakis. "Predicting shock dynamics in the presence of uncertainties", *J. of Comput. Phys.*, Vol. **217**, 260–276, 2006.

[4] J. -C. Chassaing, G. A. Gerolymos and I. Vallet. "Efficient and robust computation of 3-D compressible flows with Reynolds-stress closure", *AIAA J.*, Vol. **41**, 763–773, 2003.