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A NUMERICAL ALGORITHM FOR MIXED NONLINEAR COMPLEMENTARITY PROBLEMS AND APPLICATIONS TO CONTACT STRESS ANALYSIS

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ABSTRACT

Complementarity problems are involved in mathematical models of several applications in engineering, economy and different branches of physics, [3]. In solid mechanics, complementarity conditions are always present in contact problems. Limit Analysis and Plasticity models can also include a complementarity condition, [10]. Several publications dealing with complementarity problems have been presented, see [1, 2, 5].

In this paper we present a new feasible direction algorithm for mixed nonlinear complementarity problems. Two formulations for elastic analysis with contact based on this algorithm are also presented. One of them employs the Finite Elements Method and, the other one, the Boundary Elements Methods. We describe the numerical results with several examples.

Let be the mixed nonlinear complementarity problem, MNCP:

Find $(x, y) \in \mathbb{R}^n \times \mathbb{R}^m$ such that

$$x \geq 0, \quad F(x, y) \geq 0 \quad \text{and} \quad \begin{pmatrix} x \bullet F(x, y) \\ Q(x, y) \end{pmatrix} = 0, \tag{1}$$

where $F(x, y) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $Q(x, y) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ are smooth functions.

The present algorithm begins at an interior point, strictly verifying the inequality conditions, and generates a sequence of interior points that converges to a solution of the problem. At each iteration, a feasible direction is obtained and a line search performed, looking for a new interior point with a lower value of an appropriate potential function.

Following similar ideas as in [4], the iterative procedure is based on Newton algorithm to solve the equations in the MNCP. This one is modified in such a way that all the iterates satisfy the inequalities in the problem. Then, at the limit points the equations as well as the inequations are satisfied. Strong theoretical results were obtained, proving global convergence, as well as superlinear rate of convergence.

The analysis of contact problems has been considered by several authors, see [6-9]. Here, we present two approaches for the contact problem in linear elasticity (Signorini's problem).The first one is based on the variational formulation of Signorini's problem and uses the FEM to discretize the continuous problem. The discrete form is a finite dimensional optimization problem with linear constraints where the Karush-Kunh-Tucher optimality conditions can be formulated as a complementarity problem. The second one is based on a boundary integral formulation of Signorini's problem and employs the Boundary Element Method to discretize the boundary equations. The boundary conditions, when applied to the BEM equations, lead to a finite dimensional mixed complementarity problem. In both cases, the complementarity problem was solved with the present algorithm.

Several test problems were solved with the Finite Element Method and the Boundary Element Method and the results compared with those obtained with commercial codes. All problems were solved very efficiently and the results obtained with a very high precision. The present approach also proved to be very robust, since all the results were obtained with the same set of parameters.

The algorithm studied here is also very simple and it seems suitable to be employed to solve more sophisticated models involving contact. We mention contact with friction, nonelastic materials and dynamic problems.

REFERENCES

[1] C. Chen and O. L. Mangasarian, "A class of smoothing functions for nonlinear and mixed complementarity problems", *Comput. Optim. Appl.* **5**, 97–138. 1996

[2] R. W. Cottle, F. Giannessi and J. L. Lions, *Variational Inequalities and Complementariy Problems*, John Wiley & Sons, 1980.

[3] M. C. Ferris and J. S. Pang, "Engineering and economic applications of complementarity problems", *SIAM Review* **39**, 669–713, 1997.

[4] J. Herskovits, "A feasible directions interior point technique for nonlinear optimization", *JOTA - Journal of Optimization Theory and Applications* **99** (1), 121–146, 1998.

[5] J. Herskovits and S. R. Mazorche, A Feasible Directions Algorithm for Nonlinear Complementarity Problems and Applications in Mechanicas, *Structural and Multidisciplinary Optimization*, Springer-Verlag (submitted), 2005.

[6] N. Kikuchi and J .T. Oden, *Contact Problems in Elasticity: A Study of Variational Inequalities on Finite Element Methods*, SIAM Studies in Applied Mathematics, Philadelphia, 1988.

[7] P. D. Panagiotopoulos, "A nonlinear programming approach to the unilateral and friction boundary value problem in the theory of elasticity", *Ingenieur Archiv* **44**, 421–432, 1975.

[8] J. C. Simo, P. Wriggers and R. L. Taylor, "A perturbed lagrangian formulation for the finite element solution of contact problems", *Comp. Methods Appl. Mech. Eng.* **50**, 163–180, 1985.

[9] P. Wriggers and K. Fisher, Recent new developments in contact mechanics, *in* "4^{th} European LS-DYNA Users Conference", 2003.

[10] N. Zouain, J. Herskovits, L. A. Borges, and R. A. Feijóo, "An Iterative Algorithm For Limit Analysis With Nonlinear Yield Functions". *International Journal of Solids and Structures* **30** (10), 1397–1417, 1993.