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PROXIMAL METHODS AND ELLIPTIC REGULARIZATION FOR VARIATIONAL INEQUALITIES IN MATHEMATICAL PHYSICS

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ABSTRACT

A generalized proximal point method for solving variational inequalities with maximal monotone operators is developed. It admits a successive approximation of the feasible set and of a symmetric component of the operator as well as an inexact solving of the regularized problems. The conditions on the approximation are coordinated with the properties of finite element methods for solving problems in mathematical physics. The choice of the regularizing functional exploits a possible "reserve of monotonicity" of the operator in the variational inequality.
For the *convection-diffusion problem* as well as for the *minimal surface problem* and related variational inequalities the studied method extends the principle of elliptic regularization. A special convergence analysis shows a more qualitative convergence of the method applied to these problems than it follows from the general theory of proximal point methods.
For singularly perturbed problems, like convection-diffusion problems, elliptic regularization leads to non-perturbed elliptic problems. Therefore, the regularized auxiliary problems can be solved by standard finite element techniques. In particular, for convection-diffussion problems the boundary ore inner layers, respectively, will be accumulated gradually.
Also applications to some variational inequalities from the elasticity theory are investigated.

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