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A NEW EFFICIENT EXPLICIT NUMERICAL SCHEME FOR INTEGRATION OF DAE: APPLICATION TO CONSTITUTIVE EQUATIONS IN SOLID MECHANICS

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ABSTRACT

Systems of algebraic-differential equations (DAE) are widely used in practice in many fields of science. Mostly the solution of DAE can not be analytically obtained so the use of numerical methods becomes necessity. One of the fields, where the DAE is often employed is the field of solid mechanics, where the mathematical form of constitutive laws is mainly the system of algebraic and first order differential equations. Accurate integration of the nonlinear constitutive laws over the strain path is essential for a precise solution of any nonlinear boundary value problem in continuum mechanics, especially when additional nonlinearities are involved in the problem. Primarily this work deals with the problem of integration of nonlinear elastoplastic constitutive equations in the field of solid mechanics, thought the conclusions in this work can be used in problems from other fields, which are described with a system of algebraic and first-order differential equations.
Nowadays most commonly used schemes for integration of elastoplastic constitutive laws can be classified into the categories of implicit and explicit schemes. Implicit schemes set up the system of nonlinear algebraic equations by translation of differential equations in difference form, where all variables are written somewhere within or at the end of an increment. They are attractive, because they precisely solve algebraic equations at the end of each increment, when the use of numerical method for root finding is employed and roots of system of nonlinear equations are precisely found. One of the most popular of these is the backward-Euler scheme, which is nowadays

employed in commercial codes for finite element analysis [1]. Although most recent works preferred to use the backward-Euler scheme in the field of solid mechanics, it is difficult to implement such procedure for complex constitutive relations [2], beside the numerical iteration procedure for finding roots of system of nonlinear equations consume a lot of CPU and the dilemma arises, how precise the root should be found. On the other hand explicit methods can be generally used to implement integration for any elastoplastic constitutive law and no iteration procedure is generally requested. Yet, if the explicit integration schemes are used, the algebraic equations may not be exactly solved and consequently the stress state at the end of the increment may not fulfill the yield criterions. The error will essentially depend on the size of the strain increment. Because of discussed advantages nowadays numerous works use forward-Euler method and deal with the accuracy problems of explicit schemes [2, 3].

Objective of this paper is to present a new effective method for integration of nonlinear constitutive equations. The theoretical background of a new explicit scheme is first compared with theoretical background of backward-Euler scheme and forward-Euler scheme and discussed in details. Then the methods are employed to integrate the damage constitutive model, well known as GTN (Gurson-Tveergard-Needleman) model to study properties of presented schemes on the application. It is mathematically proven in this paper that first explicit solution of the new scheme reaches the same level of accuracy than the solution, obtained with classical backward-Euler scheme after first iteration of Newton's root finding algorithm. In new scheme such first solution can be improved with calculation of n explicit correction terms, if necessary. Again, mathematical proof is exposed that with consideration of n correction terms the accuracy of the final solution can reach the same level of accuracy than the solution, obtained with classical backward-Euler scheme after n+1 iterations. However, due to the seeming similarity with forward-Euler scheme one of the benefits of the new method is a simple implementation, because explicit equation for a specific problem can be simply derived. Another important benefit is lower computational cost, comparing to classical backward-Euler scheme, because computational procedure of the new scheme is simpler and therefore needs less computational operations to reach same level of accuracy.

Again, although the new scheme is developed for integration of constitutive equations in the field of solid mechanics, the scheme can be used in problems from other fields, which has the same mathematical form.

REFERENCES

[1] ABAQUS Version 6.6.1, 2006. User's Manual. SIMULIA, Providence, RI.

[2] Ding K.Z., Qin Q.-H., Cardew-Hall M. Substepping algorithms with stress correction for the simulation of sheet metal forming process, International Journal of Mechanical Sciences, In Press, Corrected Proof, Available online 24 March 2007

[3] Alireza V., Amirkhizi, Nemat-Nasser, S. A framework for numerical integration of crystal elasto-plastic constitutive equations compatible with explicit finite element codes, International Journal of Plasticity, Vol. **23**, pp. 1918 – 1937, (2007)