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ANALYTICAL STUDY OF THE PERFORMANCE OF NEW APPROXIMATE LOCAL *DtN* BOUNDARY CONDITIONS FOR PROLATE SPHEROIDAL-SHAPED BOUNDARIES

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ABSTRACT

We propose a new class of three-dimensional absorbing boundary conditions to be employed on prolate spheroid boundaries that are primary candidates for surrounding elongated scatterers. These conditions are constructed from a *local* approximation of the Dirichlet-to-Neumann (DtN) operator corresponding to the Helmholtz problem when expressed in elliptical coordinates. Note that when the eccentricity of the prolate spheroid tends to zero (the prolates pheroid degenerates to a sphere), these conditions coincide with the three-dimensional approximate local DtN conditions designed for spherical-shaped boundaries [1].

The construction procedure procedure for deriving these approximate local DtN boundary conditions is different for the method used in [2] and [1] for constructing these type of conditions to be employed on cylindrical and spherical boundaries. Indeed, the key ingredient of the procedure in [2], [1] is the trigonometric identities that express high order derivatives of sine and cosine functions. However, this property is not satisfied by the angular spheroidal wave functions [3]. Hence, the construction methodology we propose for the case of prolate spheroidal boundaries can be viewed as an *inverse-type* approach. More specifically, we start from a Robin-type boundary condition with unknown coefficients. Unlike the case of the spheroidal coordinates, these coefficients depend on the angles (φ, θ) of the prolate spheroidal coordinates. Such dependence is necessary to preserve the symmetry and local nature of the resulting boundary conditions. We also require that the considered condition to be an exact representation of the first modes. Consequently, we obtain the following first- and second-order boundary conditions:

$$\mathrm{DtN1}: \frac{\partial u}{\partial \xi} = \frac{\sqrt{1-e^2}}{e} R_{00}\, u$$

$$\mathrm{DtN2}: \frac{\partial u}{\partial \xi} = \frac{\sqrt{1-e^2}}{e\,(\lambda_{01}-\lambda_{00})} \left[(\lambda_{01}R_{00}-\lambda_{00}R_{01})\, u + (R_{00}-R_{01})\left(\Delta_\Gamma - (kf)^2\cos^2\varphi\right) u\right]$$

where (ξ, φ, θ) are the prolate spheroidal coordinates, Δ_Γ denotes the Laplace Beltrami, k is the wave number, f the interfocal distance of the spheroid, e its eccentricity, and a its semi-major axis. The parameter R_{mn} is defined as follows:

$$R_{mn} = \frac{R'^{(3)}_{mn}\left(eka, e^{-1}\right)}{R^{(3)}_{mn}\left(eka, e^{-1}\right)} \;,\;\; n \geq m$$

where the functions $R^{(3)}_{mn}\left(eka, e^{-1}\right)$ are the radial spheroidal wave functions of the third kind and the mn^{th} mode [3].

We analyze the effect of the wavenumber and the eccentricity values on the performance of the DtN1 and DtN2 boundary conditions when applied for solving acoustic scattering problems. Such investigation is performed analytically into two different mathematical framework depending on the frequency regime. More specifically, we assess the effect of low wavenumber values in the context of the on-surface radiation condition formulation (OSRC) [4]. The performance of these conditions in the high frequency regime is investigated using the exact Fourier representation of the solution of the acoustic scattering problem in the bounded computational domain [5]

The analysis reveals that the DtN2 boundary condition retains an excellent level of accuracy for all frequency regime and eccentricity values of the artificial boundaries. In addition, guidelines for avoiding excessive computational cost for practical applications are provided.

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