

## FAILURE INITIATION AND UNCERTAINTY INSIDE MULTI-SCALE RANDOM MEDIA

\* Robert P. Lipton <sup>1</sup>

<sup>1</sup>Louisiana State University  
Department of Mathematics  
Baton Rouge, LA 70803  
USA, lipton@math.lsu.edu,  
<http://www.math.lsu.edu/~lipton/>

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### ABSTRACT

Engineering materials such as fiber reinforced composites and steel reinforced concrete exhibit superior properties that follow directly from their multi-scale structure. On the other hand the process of making composite structures involves several steps and introduces uncertainty at the level of the composite micro-geometry. This uncertainty is manifested in the variability of the strength and overall stiffness properties of composite structures. These uncertainties are unavoidable and include local variations in micro-geometry due to processing as well as the presence of defects generated during service.

From the perspective of failure initiation it is important to discover new ways to link the location of high stress zones with the local statistics describing the microgeometry inside random heterogeneous media. In this talk we discuss the problem of resolving the location of high stress zones inside composite media using coarse-grained field measurements together with statistical descriptions of the microstructure. We show how to apply the statistics and coarse-grained measurements to assess the extent of high stress zones inside heterogeneous structures under suitable assumptions on the size of the heterogeneities. The numerical methods presented here demonstrate how macrostress modulation functions [1], [2], [3] are used to relate the statistics of the microstructure to the local field behavior inside the composite media.

### REFERENCES

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