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# A stable mixed discontinuous/continuous finite element pair suited to large scale ocean simulations.

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## ABSTRACT

We introduce a new mixed discontinuous/continuous Galerkin finite element for solving the 2- and 3-dimensional wave equations and equations of incompressible flow. The element, which we refer to as $P1_{DG}$-P2, uses discontinuous piecewise linear functions for velocity and continuous piecewise quadratic functions for pressure. The aim of introducing the mixed formulation is to produce a new flexible element choice for triangular and tetrahedral meshes which satisfies the LBB stability condition and hence has no spurious zero-energy modes. The advantage of this particular element choice is that the mass matrix for velocity is block diagonal so it can be trivially inverted; also it allows the order of the pressure to be increased to quadratic whilst maintaining LBB stability which has benefits in geophysical applications when Coriolis is present.

We give a normal mode analysis of the semi-discrete wave equation in one dimension which shows that the element pair is stable in one dimension, and demonstrate that the element is stable with numerical integrations of the wave equation in two dimensions, an analysis of the resultant discrete Laplace operator in two and three dimensions on various meshes which shows that the element pair does not have any spurious modes. We provide convergence tests for the element pair which confirm that the element is stable since the convergence rate of the numerical solution is quadratic.

We then demonstrate the applicability of this element to the simulation of ocean flow problems. The stability and accuracy of this method for incompressible flows is illustrated by solving problems such as the wind driven cavity and lock exchange problems while the advantage in representing geostrophic balance afforded by the quadratic pressure representation is demonstrated by the simulation of rotating problems such as trapped Kelvin waves in shallow water.