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INVERSE HOMOGENIZATION AND OPTIMAL DESIGN OF COMPOSITE STRUCTURES FOR STRENGTH AND STIFFNESS

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ABSTRACT

It is now well known that homogenization theory is an effective tool for the design of composites for optimal structural compliance and natural frequency. On the other hand the more delicate problem of designing composites subject to local stress constraints has received attention only recently in [1], [2] and [3]. In those treatments constraints on the mean square stress are considered.

In this work we treat the composite design problem in the presence of point wise constraints on the invariants of the local stress and strain tensors inside the composite structure. Our method is based on new multi-scale quantities for measuring the stress transfer between macroscopic to microscopic length scales. These quantities are able to capture the amplification of the local stress due to the composite microstructure [4], [5] and [6]. We illustrate the design method for two different examples. The first example illustrates a numerical method for the design of optimally graded fiber reinforced shafts. Here the reinforcement fibers have circular cross sections and their diameters can change across the shaft cross section. For this example the optimal design delivers a fiber reinforced shaft with sufficient overall stiffness while at the same time keeping the magnitude of the local stress at each point below a prescribed value. The second example is carried out in the context of plane strain loading. Here we recover a novel class of designs made from locally layered media for minimum compliance subject to constraints on the magnitude of the local stress.

The numerical examples are seen to agree with intuition and automatically place the more compliant material in the neighborhood of stress risers associated with abrupt changes in boundary loading and reentrant corners, see [4], [5], and [6].

REFERENCES

[1] P. Duysinx and M. P. Bendsöe. “Topology optimization of continuum structures with local stress constraints”. *Int. J. Num. Meths.Eng.*, Vol. **43**, 1453–1478, 1998.

[2] R. Lipton. “Design of functionally graded composite structures in the presence of stress constraints”. *Int. J. Solids and Structures* Vol. **39**, 2575–2586, 2002.

[3] G. Allaire, F. Jouve and H. Mallot. “Topology optimization for minimum stress design with the homogenization method”. *Struct. Multidisc. Optim.* Vol. **28**, 87–98, 2004.

[4] R. Lipton and M. Stuebner. “Inverse homogenization and design of microstructure for point wise stress control”. *The Quarterly Journal of Mechanics and Applied Mathematics* Vol. **59**, 139–161, 2005.

[5] R. Lipton and M. Stuebner. “Optimization of composite structures subject to local stress constraints”. *Computer Methods in Applied Mechanics & Engineering* Vol. **196**, 66–75, 2006.

[6] R. Lipton and M. Stuebner. “Optimal design of composite structures for strength and stiffness: an inverse homogenization approach”. *Struct. Multidisc. Optim.* Vol. **33**, 351–362, 2007.