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A NEW TIMOSHENKO BEAM MODEL BASED ON A MODIFIED COUPLE STRESS THEORY

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ABSTRACT

Thin beams have found important applications in micro- and nano-scale measurements (e.g., in biosensors and atomic force microscopes). Lacking a material length scale parameter, the classical beam theories cannot interpret the size effect observed in bending tests at micron and nanometer scales. Hence, these classical beam models need to be extended to incorporate material microstructural features [1,2]. Higher-order (nonlocal) continuum theories contain additional material constants and are capable of effectively explaining various size effects. Therefore, one such higher-order theory, known as the modified couple stress theory [3,4], is employed in this study to develop a new beam model of the Timoshenko type. This theory has recently been used to provide a Bernoulli-Euler beam model that contains a material length scale parameter [1].

The governing equations and boundary conditions for the Timoshenko beam are obtained by using the Hamilton's principle. The strain energy density function of the beam consists of two parts: one is a conjugated pair of stress and strain tensors, and the other is that of couple stress and curvature tensors. A new internal couple, as the resultant of the couple stress, is introduced in the model. This internal couple may be regarded as an additional controlling parameter in designing micro-scale beams.

To illustrate the newly developed Timoshenko beam model, a simply supported beam is analyzed. The numerical results reveal that the bending rigidity of the beam predicted by the new model is stiffer than that predicted by the classical Timoshenko beam model. In addition, it is found that the difference between the current and the classical beam models is significantly large when the beam is very thin (on the order of 10 μm here) but is diminished as the beam thickness increases to about 100 μm, thereby capturing the size effect at the micron scale.

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