8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Boundary Conditions for Combined Compact Difference Schemes with High Resolution

***Kazuya Matsuoka[1] and Katsuya Ishii[2]**

[1]Nagoya University,
Dept. Info. Engng., Furo-cho,
Chikusa-ku,
Nagoya 464-8603, Japan
matsuoka@hpc.itc.nagoya-u.ac.jp

[2]Nagoya University,
ITC, Furo-cho, Chikusa-ku,
Nagoya 464-8601, Japan
ishii@itc.nagoya-u.ac.jp

Key Words: *Combined Compact Difference scheme, boundary condition, stability analysis.*

ABSTRACT

Various phenomena with different spatial and temporal scales are found in many flows, turbulent fluid flows and aeroacoustic noise being common examples. Direct numerical simulations of these flows require high resolution and high accuracy. One candidate of these numerical scheme is the Compact Difference (sp-CD) schemes with spectral-like resolution proposed by Lele[1] . The CD schemes have been used in many studies of fluid phenomena. However, these numerical difference schemes must be used at a number of points near the boundary. Lele discussed the stability in the semi-discrete form of CD schemes for a 1-D advection equation and showed that the sp-CD schemes are numerically stable. Carpenter et al.[2] assessed the stability characteristics of various 4th-order and 6th-order CD schemes for the semi-discrete initial boundary value problem. It was shown that the stability of CD schemes are sensitive to choice the difference equation at the boundary points, that is, the accuracy of the scheme and the number of stencils used in the scheme.

The Combined Compact Difference (CCD) scheme was proposed by Chu and Fan[3]. Higher derivatives at 3-point stencil are used in the CCD schemes. The CCD schemes have higher resolution than CD schemes with the same accuracy. Nihei and Ishii[4] developed the Combined Compact Difference (sp-CCD) scheme with spectral-like resolution for solving the shallow water equations. We investigated the stability property of the sp-CCD scheme for

1-D advection-diffusion equation. It was shown that the boundary equation is important to the stability of CCD schemes[5]. In particular the stability of CCD schemes is sensitive to choice the order of accuracy of each derivatives included in the boundary scheme.

The CCD schemes have been derived for uniform grid systems. For applications of these CCD schemes to problems with complex geometries and boundary conditions, the usual approach is to use a mapping from a nonuniform grid system to a uniform grid system and apply these schemes directly on the mapped coordinate. However, to solve problems with too complex geometries and boundary conditions, the use of body fitted grid systems is complicated and difficult.

In this study, we propose a CCD scheme for the irregular boundary condition in which the boundary is located between regular grid points. We investigate the Dirichlet and/or Neumann boundary conditions in the new CCD scheme. We assume that the regular grid system has uniform spacing . The boundary is located at the distance L from the inner boundary grid point that is adjacent to the boundary. The new CCD scheme is constructed by using the value L. We investigate the stability of the fully discretized equation used the proposed scheme and Runge-Kutta method for a 1-D advection-diffusion problem because the stability of finite difference schemes is sensitive to the boundary equation. In addition, we discuss the extension of the new scheme to multidimensional problems of hyperbolic systems.

REFERENCES

[1] Sanjiva K. Lele: Compact Finite Difference Schemes with Spectral-like Resolution, J. Comput. Phys. **103**, 16-42 (1992)

[2] Mark H. Carpenter, David Gottlieb, and Saul Abarbanel: The stability of numerical boundary treatments. for compact high-order finite-difference schemes. J. Comput. Phys., **108**, 272–295 (1993)

[3] Peter C. Chu and Chenwu Fan: A Three-Point Combined Compact Difference Scheme, J. Comput. Phys. **140**, 370-399 (1998)

[4] Tomonori Nihei, Katsuya Ishii: A fast solver of the shallow water equations on a sphere using a combined compact difference scheme, J. Comput. Phys. **187**, 639-659 (2003)

[5] Kazuya Matsuoka and Katsuya Ishii: Stability Analysis of Combined Compact Difference Method, Theoretical and Applied Mechanics Japan **55,** 239-246 (2006)