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PARALLEL COMPUTATIONAL ALGORITHM OF THE SOLUTION OF DYNAMIC PROBLEMS FOR ELASTIC–PLASTIC AND GRANULAR MATERIALS

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ABSTRACT

A process of waves propagation in elastic-plastic and granular materials under small strains is described on the basis of rheological model, which in edition to traditional elastic and plastic elements includes a new element – rigid contact [1]. This element describes the behaviour of perfectly granular material with rigid particles. The Mises–Schleicher destruction criterion is used to describe the granularity of medium and the Mises yield criterion is used to determine the plasticity of grains. It is shown that the stress tensor can be obtained as a projection onto the cone of admissible stresses of the conditional-stress tensor which is determined from Hook's law over elastic part of the strain tensor. Its plastic part can be found from the associative flow rule. The closed mathematical model consists of the system of equations of motion, kinematic and constitutive relationships.

Let U and V be vector–functions such that the former consists of the velocity vector components and nonzero components of the conditional stress tensor and the latter consists of the velocities and the real stresses. In terms of these functions the model is transformed to the variational inequality

$$(\widetilde{V} - V)\biggl(A\,U_{,t} - \sum_{i=1}^{n} B^i\,V_{,x_i} - G\biggr) \ge 0, \quad V,\,\widetilde{V} \in F; \qquad V = \lambda\,U + (1-\lambda)\,U^{\pi},$$

where $\widetilde{V}$ is the varied vector, A and B^i are the symmetric matrices, G is a given vector–function, n is the spatial dimension of the problem, the subscripts denote the partial derivatives with respect to time and spatial variables, superscript π denotes the projection onto the Mises–Schleicher cone, $\lambda \in (0,1]$ is the regularization parameter, F is the Mises cylinder. The algorithm used for the numerical implementation of variational inequality is explicit in time and is constructed as follows. First, the problem of deformation of a heteromodular elastic material is solved at each time level. Next, the resulting solution is corrected so as to take into account plastic properties of the material. The boundary conditions can be given in the terms of velocities as well as in stresses. Numerical algorithm is based on a combination of the space-variable splitting method and the special procedure of the stresses correction taking into account irreversible strains [2]. On each stage of the splitting method 1D problems are solved by means of the monotone ENO–scheme with limit reconstruction of the solution.

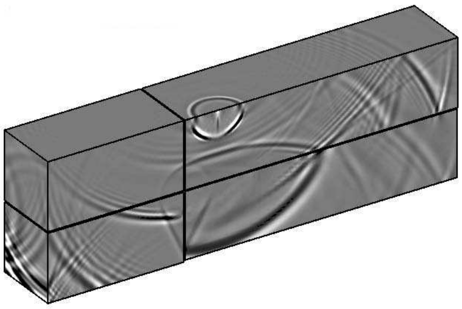

Figure 1. Lamb problem for medium with rigid inclusion

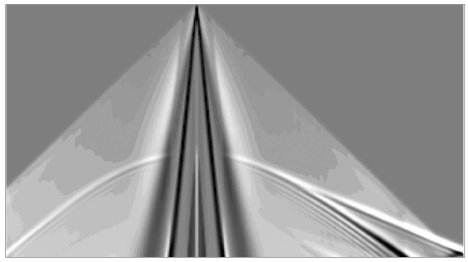

Figure 2. Seismogram of vertical displacement

This algorithm is realized as a parallel program system in Fortran–95 on the basis of SPMD technology with the use of MPI library. Program system allows to simulate the propagation of elastic-plastic waves produced by mechanical impacts in a body, aggregated of heterogeneous blocks with curvilinear boundaries. It may be applied for the solution of direct seismic problems taking into account complicated mechanical properties of geomaterials. The system consists of the front-end processor program, the main program of velocities and stresses computation, the subprograms for the realization of boundary conditions and pasting together conditions in interior boundaries, and the back-end processor program. The front-end processor is intended for the constructing of curvilinear grids in blocks, for the initial data preparation in a packed form and for their uniform distribution between parallel computational nodes. The main program on each node of cluster makes a similar computations consisting of mutually coordinated step-by-step realization of the space-variable splitting method. Data interchange between the processes is carried out at the stage of limit reconstruction of the solution of 1D systems. The back-end processor performs special resampling down of data bulks to lower a time of traffic along the global network and a time of the results graphic processing with the help of personal computer software.

Some model dynamic problems for elastic, elastic-plastic and granular materials were solved in 2D and 3D settings by means of described program system at the cluster of MVS series [2]. In Fig. 1 one can see the level surfaces of vertical normal stress in spatial Lamb problem for two-layer mass of elastic medium. The elasticity parameters of compact ground are assigned in the upper layer and in the left part of the lower one, the parameters of strong rock – in the right part of the lower layer. The seismogram, describing a behaviour of vertical component of the displacement vector depending on the time, is represented in Fig. 2. Time-distance plots of head waves and refractions are rectilinear segments, time-distance plots of reflections are piecewise-parabolic splines, changing their curvature at the points of transition from the reflection from interface to the reflection from angle. Computations were carried out on 68 processors, each of which processes the grid including $50 \times 50 \times 50$ meshes.

Numerical results suggest that the used shock-capturing method has a satisfactory accuracy. Judging from fulfilled computations, parallel computing systems possess of essential advantages as compared with personal computers if dimensionalities of spatial grids in solving problems are sufficiently large.

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