8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

THE CRITICAL LEFT AND RIGHT EIGENVECTORS EXTRACTED FROM THE *LDU*-DECOMPOSED NON-SYMMETRIC JACOBIAN MATRIX

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KEY WORDS: Bifurcation, Non-symmetric Jacobian Matrix, Left and Right Eigenvectors, LDU-Decomposition, Eigenanalysis-free.

ABSTRACT

The writers have already proposed an eigenanalysis-free procedure [1, 2, 3, 4] to approximately extract the critical eigenvector(s) and eigenvalue, only from the large-scale $\boldsymbol{LDL^T}$-decomposed symmetric stiffness matrix in the vicinity of a stability point. Any eigensolver, for example, subspace method, is not needed and only $\boldsymbol{LDL^T}$-decomposer and equation solver are necessary. For a symmetric stiffness matrix, there is no difference between the left and right eigenvectors.

In this WCCM08-paper, the proposed eigenanalysis-free idea will be extended to the $\boldsymbol{LDU}$-decomposed non-symmetric Jacobian matrix;

$$\boldsymbol{J} = \boldsymbol{LDU}^T$$

with

$$\boldsymbol{J} \neq \boldsymbol{J}^T$$

for which the eigenvalues may be complex, and the left and right eigenvectors, satisfying;

$$\phi^T \boldsymbol{J} = \lambda \phi^T$$

and

$$\boldsymbol{J}\theta = \lambda\theta$$

are in general not identical to each other.

There are two points [5]: Firstly, the engineering interpretation and roll assignment for the critical left and right eigenvectors for the non-symmetric Jacobian matrix will be

clarified in this paper from the view point of the mathematical bifurcation theory. It is mentioned how the critical left and right eigenvectors will be used in bifurcation analysis. Secondly, it will be shown that both of the critical right and left eigenvectors may be extracted near a singular point, from the unit lower triangular matrix ***L*** and the unit upper triangular matrix ***U***, respectively, when the critical diagonal d_m (m: index number to identify the critical diagonal) in ***D*** is identified near the singular point. The linear-algebraic background necessary for this extended idea is the core part of this WCCM- paper. The idea is purely mathematical and independent of the physical background of the problem to be solved, so that it may be applied to any non-symmetric Jacobian matrices, which will be singular in engineering stability problems.

Numerical stability examples are computed for verification and discussion and may include problems with following-forces, contact and pontoon stability [6] in structural engineering and non-associated flow rule for geo-materials [5].

REFERENCES

[1] Fujii, F. and Noguchi, H.
The Buckling Mode Extracted from the LDL^T-Decomposed Large-Order Stiffness Matrix,
Communications in Numerical Methods in Engineering, Vol.18, Issue 7, 459-467 (2002)

[2] Noguchi, H. and Fujii, F.
Eigenvector-free Indicator, Pinpointing and Branch-Switching for Bifurcation
Communications in Numerical Methods in Engineering, Vol.19, Issue 6, 445-457 (2003)

[3] Fujii, F. and Noguchi, H.
Eigenvector-free Procedures in Computational Bifurcation Analysis,
ICCES'03, Keynote Lecture(2003)

[4] Noguchi, H., Yatake, S. and Fujii, F.
Buckling Mode Extracted from Iterative Solution Procedures for Stiffness Equations
Second MIT Conference on Computational Fluid and Solid Mechanics (2003)

[5] Fujii, F. and Yamakawa,Y.
Left and Right Eigenvectors Extracted from the ***LDU***-Decomposed Jacobian Matrix in Stability Problems
APCOM2007, Kyoto, CD-ROM Proceedings, December 2007

[6] Petersen, C.
Statik und Stabilität der Baukonstruktionen
Friedr.Vieweg & Sohn, 2. durchgesehene Auflage,1982,Braunschweig/Wiesbaden