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NUMERICAL COMPUTATIONS IN THE PROBLEMS OF THE LIMIT EQUILIBRIUM OF MATERIALS WITH DIFFERENT STRENGTHS

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ABSTRACT

Phenomenological models of materials having different resistance to tension and compression are constructed using the rheological method supplemented by a new element – rigid contact. Questions of the solvability of static boundary–value problems are considered within the framework of regularized model. Generalization of static and kinematic theorems of the theory of the limit state is given. The iterative algorithm is suggested for numerical realization of this model on the basis of a finite elements approximation.

Rheological models of materials, having different resistance to tension and compression (granular and porous media: soils, rocks, concretes, graphites, etc.), are constructed using an auxiliary element – rigid contact [1]. The simplest model, taking into account the connectivity of material, is illustrated by rheological diagram in Fig. 1. Such a material does not deform under compressive stresses or tensile stresses less than the cohesion coefficient σ_0 (the yield point of plastic element). The attainment of the value of σ_0 corresponds to the limit equilibrium state, in which the strain can be an arbitrary positive quantity. Stresses above this limit are impossible. The constitutive equations of uniaxial deformation for monotonic loading without unloading allow the potential representation

$$\sigma \in \partial\Phi(\varepsilon), \qquad \varepsilon \in \partial\Psi(\sigma). \tag{1}$$

Here $\Phi = \sigma_0\,\varepsilon + \delta_C(\varepsilon)$ and $\Psi = \delta_K(\sigma - \sigma_0)$ are the potentials of stresses and strains; indicator functions, vanishing on the cones $C = \{\varepsilon \geq 0\}$ and $K = \{\sigma \leq 0\}$ and infinite outside these cones, are denoted by δ; ∂ serves for the notation of the subdifferential of a convex function. The extension to the case of three-dimensional stress–strain state is constructed on the basis of inclusions (1). For this a symmetrical cohesion tensor σ_0, a convex and closed cone C with vertex at the origin of six-dimensional space of the strain tensors or analogous cone K in the space of the stress tensors are given.

A model of granular material with elastic grains serves as the regularization of the last one. The rheological diagram of this model is presented in Fig. 1, where a and b are tensors of elastic coefficients of the springs. Closed mathematical model for the description of strain–stress state of a medium is formed by the constitutive equations in the form (1), supplemented by the conditions of equilibrium

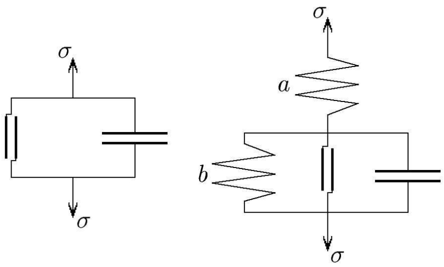

Figure 1. Rheological diagrams

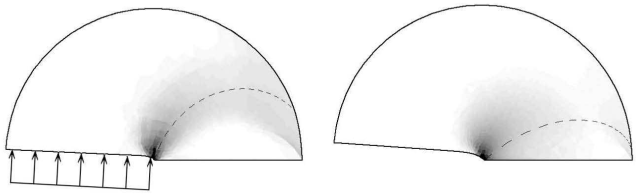

Figure 2. The intensity of shear in a sample (two materials)

and kinematic constraints. In final form the problem leads to minimization of the next integrals in terms of displacement vector and stress tensor

$$I(u) = \int\limits_{\Omega} \Big(\Phi\big(\varepsilon(u)\big) - f \cdot u\Big)\, d\Omega - \int\limits_{\Gamma_\sigma} q \cdot u \, d\Gamma, \qquad J(\sigma) = \int\limits_{\Omega} \Psi(\sigma)\, d\Omega \tag{2}$$

in linear subspace $u \in U$ of the Sobolev space $H^1(\Omega)$ of distributions, which satisfy the homogeneous boundary condition on Γ_u and in affine subspace $\sigma \in \Sigma$ of the tensor functions from $L_2(\Omega)$, for which the equilibrium equations and boundary conditions on Γ_σ "in terms of stresses" are satisfied.

According to the limit model, in which $a \to \infty$, the domain Ω splits into two parts – a rigid domain, where the material does not deform, and a domain of non-zero deformation. The applied external load (f, q) is said to be safe, if there is no the deformation zone. In this case the displacement vector is equal to zero or it represents the motion of the medium as a rigid body. Safe loads form the convex and closed set S in the Cartesian product of the spaces $L_2(\Omega)$ and $L_2(\Gamma_\sigma)$. The next criterion can be proved [2]: the acting load belongs to S if and only if $\Sigma_K \neq \emptyset$. On the other hand it is equivalent to the next inequality for each $\tilde{u} \in U_C$

$$\int\limits_{\Omega} \big(\sigma_0 : \varepsilon(\tilde{u}) - f \cdot \tilde{u}\big)\, d\Omega - \int\limits_{\Gamma_\sigma} q \cdot \tilde{u}\, d\Gamma \geq 0. \tag{3}$$

Boundary points of the set S correspond to the limit loads. One can determine the safety factors – non-negative numbers m_f and m_q for which the load $(m_f f, m_q q)$ is limiting.

The criteria obtained above represent static and kinematic theorems of the theory of limit state. They provide us with a simple method of estimating the safety factors of the load. As an example we will consider the plane strain state of homogeneous cylindrical sample with radial notch, whose sides are loaded with a pressure. We consider a specific class of fields of displacements with certain lines of the localization of deformation and show that the line of the extremum of $I(u)$ can be logarithmic spiral only (the hypothesis of Drucker and Prager). We apply the combination of a finite elements technology with the method of initial stresses to solve such a problem numerically. Some results are represented in Fig. 2, where the broken lines correspond to logarithmic spirals.

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