8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A forward model for Optical Diffuse Tomography based on a cardinal Splines

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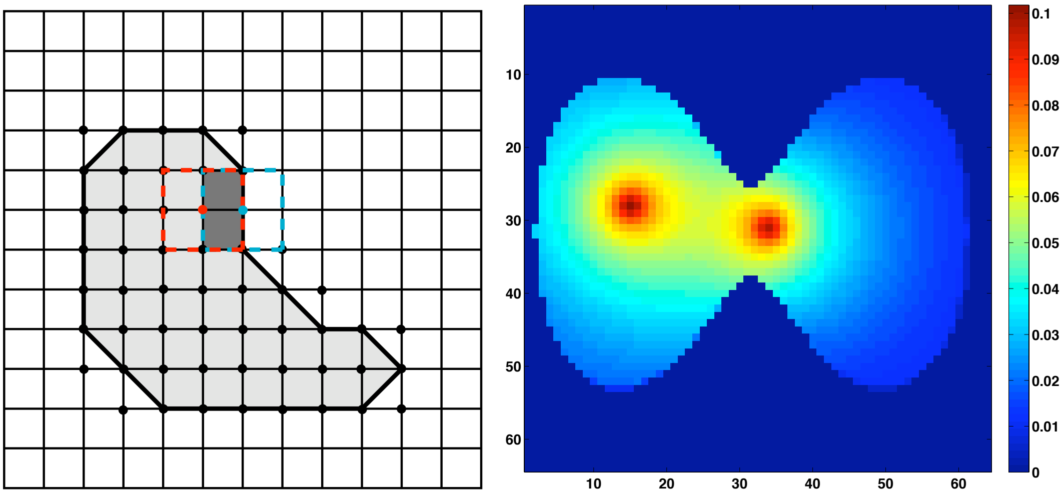
Key Words: *Applications, Mesh-free FEM, Optical Diffuse Tomography, Splines*

ABSTRACT

In Optical Diffuse Tomography (ODT), one is interested in recovering optical properties of a diffusive medium, as well as positions and intensities of some internal light sources, from peripheral light measurements [1]. Typically, the medium is a biological tissue where the propagation of light is strongly affected by absorption and scattering. In those media, and for the wavelength window of interest (near infra-red), photon transport can be aptly modeled using a diffusion equation with Robin boundary conditions [2]. A key step to solve the inverse problem (i.e. reconstructing the light sources from the boundary measurements) is to build a fast and accurate forward model that solves the diffusion equation. An appropriate forward model needs to handle rigorously arbitrary geometries and boundary conditions, and standard FEM packages can be used for that purpose [3, 4]. However, these methods require to mesh the domain of interest, which is impractical on a routine basis. The other downside of the FEM is that the basis functions are often not compatible with the ones used for solving the inverse problem, which typically have less degrees of freedom.

To overcome these limitations, we propose a spline-based alternative that we believe to be better suited for the ODT inverse problem. Our approach differs from standard FEM in two aspects. First, we have a mesh-free method, in which the discretization of the variational formulation is done by using linear B-splines (hat functions) on a regular cartesian grid. Second, we describe the boundaries using only line segments that link a grid point to one of its 8 neighbors on the grid. While preserving a good approximation of the boundary, this scheme yields a finite number of geometrical configurations of the boundary with respect to the basis functions, which considerably simplifies the generation of the stiffness matrix because the integrals that are involved can be readily pre-computed. In addition, the Robin boundary conditions are implicitly satisfied by incorporating the appropriate contour integral



(a) Example of a cartesian grid with a discretized domain (light grey) and its discretized boundary (thick black segments). In blue and red, the support of two basis functions. In dark grey, the intersection of their supports with the domain (used for the integrals computation). The black dots label the bivariate linear B-splines appearing in the stiffness matrix.

(b) Forward model solved for 2 point sources placed in an oval domain.

in the calculation of the stiffness matrix. In this work, we have implemented this method in 2D, but it can be extended to 3D as well. Figure a) illustrates the principle of our approach, and Figure b) presents the result of the forward model solved for two point sources placed in an oval domain. Finally, a significant advantage to use a mesh-free method based on B-splines is that spline spaces admit a multi-resolution decomposition, which enables to apply wavelet techniques or multigrid approaches for subsequent reconstruction. As a consequence, our custom mesh-free spline-based FEM can be coupled to a reconstruction algorithm for ODT.

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