

## UNCERTAIN RESPONSE OF A SUSPENDED BEAM UNDER A MOVING OSCILLATOR

J.D. YAU<sup>1</sup> and L. Frýba<sup>2</sup>

<sup>1</sup> Associate professor  
 Department of Architecture, Tamkang  
 University, 10620 Taipei, Taiwan.  
 jdyau@mail.tku.edu.tw

<sup>2</sup> Professor  
 Institute of Theoretical and Applied Mechanics,  
 v.v.i., ASCR, Prague, Czech Republic  
 fryba@itam.cas.cz

**Key Words:** *Suspended beam, Vehicle-bridge interaction, Rail irregularity, Vibration.*

### ABSTRACT

In the past several years, random vibration of bridges due to moving loads has been investigated by theoretical formulation and finite element modelling [1-2]. This paper is aimed at studying the effect of track random irregularities on the dynamic response of a suspended beam traversed by an oscillator running over the beam with various constant speeds. The dynamic behaviour of the suspended beam investigated herein is limited to the vertical plane vibration of a single-span beam with two-hinged supports. The oscillator is modelled as a lumped mass supported by a spring-dashpot system.

To analyze the dynamic interaction responses of the vehicle-bridge model, the beam deflection is expressed as a series of admissible shape functions and then solved by Galerkin's method so that the coupled equations containing the moving oscillator are transformed into a set of time-dependent coupled differential equations in the generalized system. With all the coupled terms and time-dependent coefficients treated as pseudo forces, the coupled differential equations in terms of generalized displacements are reduced to a set of uncoupled equations of motion and then solved by Newmark's  $\beta$  method in the time domain using a rigorous incremental-iterative procedure [3] based on the concept of *predictor, corrector, and equilibrium-checking*.

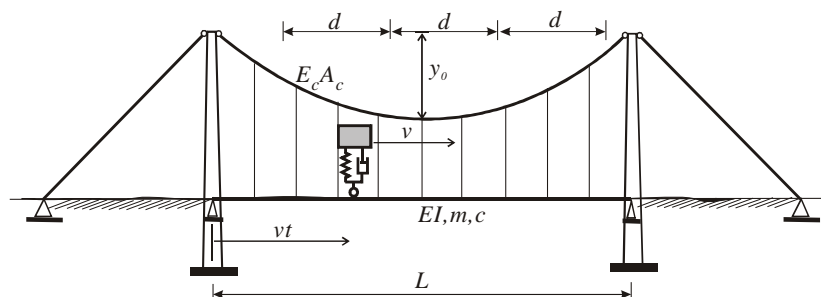


Fig. 1 Model of a suspended beam subjected to a moving oscillator

To take into account the random nature and characteristics of track irregularities on the dynamic interaction between the suspended beam and the moving oscillator, the vertical profile of track irregularities for the simulation of track geometry variations using the following power

spectrum density (PSD) function [4]

$$S(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}$$

where  $\Omega$  = spatial frequency, and  $A_v$ ,  $\Omega_r$ , and  $\Omega_c$  are relevant parameters.

With the different track classes 4, 5, and 6 designed by *Federal Railroad Administration* (FRA, USA, class 6 indicates the best quality with  $A_v = 1.5 \times 10^{-6}$  m) for track irregularities [4], the maximum acceleration responses of the suspended beam traversed by the moving oscillator with various constant speeds have been computed. Ranging the moving speeds from 0 to 280 km/h for the moving oscillator, the largest maximum standard deviations of acceleration for the beam have been plotted in Fig. 2. The results indicate that the maximum standard deviation of beam accelerations may occur at the one or three quarters along the beam span. Moreover, Fig. 2 shows that the effect of different irregular profiles of track may not produce too much difference on standard deviation for the maximum acceleration response of the suspended beam.

However, the maximum accelerations of the oscillator running on the suspended beam are strongly dependent on the nature of track irregularities, as indicated in Fig. 3. It is concluded that the track irregularity is one of an important factors that can determine the riding comfort and running safety of moving trains travelling over a suspension bridge.

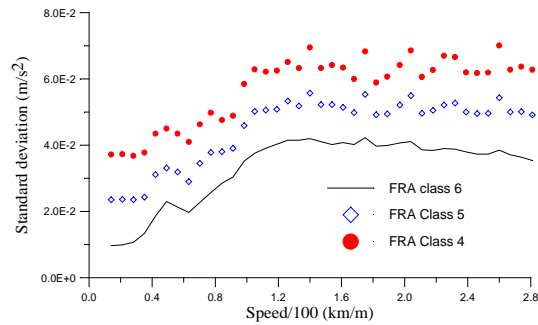
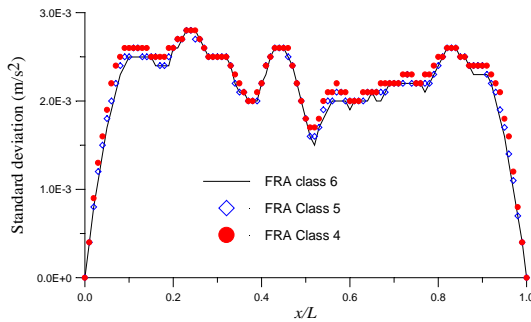


Fig. 2 Standard deviation of beam acceleration Fig. 3 Maximum acceleration of oscillator

## REFERENCES

- [1] L. Fryba, Non-stationary response of a beam to a moving random force, *Journal of Sound and Vibration*, 1976; 46:323–38.
- [2] L. Fryba, S. Nakagiri S, N. Yoshikawa, Stochastic finite elements for a beam on a random foundation with uncertain damping under a moving force, *Journal of Sound and Vibration* 1993; 163:31–45.
- [3] Y.B. Yang, S.R. Kuo, *Theory and Analysis of Nonlinear Framed Structures*, Prentice Hall, Singapore, 1994.
- [4] Y.B. Yang, J.D. Yau, Y.S. Wu, *Vehicle-Bridge Interaction Dynamics*, World Scientific, Singapore, 2004.