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TWO-SCALE ADAPTIVE FEM MODELING OF NONLINEAR HETEROGENEOUS MATERIALS

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ABSTRACT

We consider structural components made of heterogeneous material that globally possesses either elastic or inelastic properties. We assume that the micro-structure of such material is known a'priori and modeling by the computational homogenization, involving at least two scales, may be used. The adaptive FEM modeling was already used for multi-scale modeling of elastic bodies [4]. We propose to extend this approach for inelastic materials.

The macro-scale problem consist of finding the inelastic strain ($\boldsymbol{\epsilon}^*$) and displacement ($\boldsymbol{u}$) fields as well as the resulting small strains ($\boldsymbol{\epsilon}$) and stresses ($\boldsymbol{\sigma}$) that satisfy the standard equations of the solid mechanics. Inelastic strains and the effective material properties are determined for each Gauss point on the basis of solution of a micro-scale problem defined over representative volume elements (RVE). We assume that the micro-structure may be also analyzed by a continuum model with piecewise continuous material parameters and possible debonding of the components.

Despite the homogenization error the FEM discretization of problems at both scales results in approximation errors. Therefore, we make use of a'posteriori error estimates and adaptive mesh refinement [2] in order to obtain reliable results with the smallest possible number of degrees of freedom. While adaptation at the fine scale may be done in a standard way, the coarse scale mesh refinement is more challenging since mapping of the solution between old and new RVE would be a cumbersome task.

Despite a reliable a'posteriori error estimation the following three factors contributing to adaptation at the macro-scale are important for the efficiency of the approach: solution mapping after mesh refinement or unrefinement, adaptive distribution of RVEs and localized multiscale approach.

We propose to make use of a fixed set of points that are attributed to RVEs. Similar approach was already proposed in [1] and independently in paper [3]. Thus, the macro-scale (effective) quantities like the tangent tensor of material parameters are approximated by continuous or piecewise continuous functions on the basis of the point-wise values determined at the fixed points. This way despite of having well posedness of the macro-scale problem we avoid the necessity of ambiguous transfer of the problems from the old RVE grid to the new one.

In order to provide an optimal distribution of the RVEs they should be positioned in such a way that the approximation error of the macro-scale quantities is as small as possible. Naturally, this optimal distribution of RVEs may vary during the loading process. Therefore, a rough initial elastic analysis of the whole loading history, based on only one RVE is used to predict the optimal positions of the RVEs.

Finally, we assume that the multiscale analysis may be performed only at selected subregions resulting in a hybrid approach. Therefore, one should asses where in the considered domain the multiscale approach is necessary. It may be done on the basis of a'priori knowledge, e.g. the area surrounding the tip of a propagating crack is suitable for multiscale modeling, or by homogenization error estimate [4].

We verified the proposed approach on a 1D composite bar benchmark problem (Fig.1). The bar was in the uniaxial stress state and comprised of two materials with different properties (varying periodically). One of the components underwent elastic-plastic deformation (with linear strain hardening), while the other remained in the elastic range. We applied three types of the error estimates (hierarchical, explicit residual and recovery) and used them to perform *h*-refinements of the FEM mesh. Numerical tests indicate that the first error estimate gives the best rate of convergence (Fig.2).

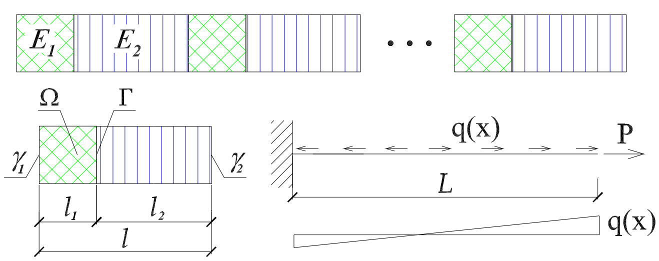

Fig.1; Bar model

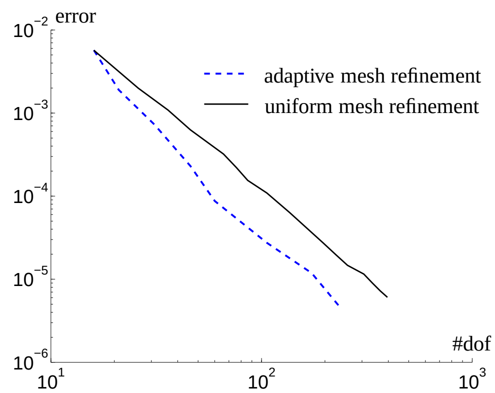

Fig.2; Convergence of adaptively and uniformly refined meshes

The adaptive approach not only enables effective numerical analysis of two-scale problems but also delivers reliable, error controlled results. A suitable adaptively selected distribution of the fixed (mesh independent) points that are attributed to RVEs is important for efficiency of the adaptive FEM analysis of the inelastic problems. Further plans include two-scale modeling of 2D and 3D problems using not only h but also the most efficient hp-adaptive mesh refinement.

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